

NAME \_\_\_\_\_

10-DIGIT PUID \_\_\_\_\_

RECITATION INSTRUCTOR \_\_\_\_\_

RECITATION TIME \_\_\_\_\_

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TOTAL	/100

## DIRECTIONS

- Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
- The test has four (4) pages, including this one.
- Write your answers in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses in the left hand margin.
- No books, notes, calculators, or any electronic devices may be used on this test.

- (12) 1. Determine whether the following statements are true or false for any series  $\sum_{n=1}^{\infty} a_n$  and

$\sum_{n=1}^{\infty} b_n$ . (Circle T or F. You do not need to show work).

(a) If  $0 < a_n < b_n$  for all  $n$  and  $\sum_{n=1}^{\infty} b_n$  diverges, then  $\sum_{n=1}^{\infty} a_n$  diverges. T F

(b) If  $0 < a_n$  for all  $n$  and  $\lim_{n \rightarrow \infty} \frac{a_n}{\frac{1}{n^3}} = 2$ , then  $\sum_{n=1}^{\infty} a_n$  converges. T F

(c) If  $\sum_{n=1}^{\infty} a_n$  diverges, then  $\sum_{n=1}^{\infty} |a_n|$  diverges. T F

- (12) 2. Determine whether each of the following series is convergent or divergent. (You do not need to show work).

(a)  $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

(b)  $\sum_{n=1}^{\infty} \frac{\pi^n}{3^{n+1}}$

(c)  $\sum_{n=0}^{\infty} \left( \frac{1}{n+1} - \frac{1}{n+3} \right)$

(30) 3. Determine whether each series is convergent or divergent. You must verify that the conditions of the test are satisfied and write your conclusion in the small box.

(a) 
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}$$

Show all necessary work here:

By the \_\_\_\_\_ test, the series is \_\_\_\_\_

(b) 
$$\sum_{n=1}^{\infty} \frac{n}{(n+1)2^{n-1}}$$

Show all necessary work here:

By the \_\_\_\_\_ test, the series is \_\_\_\_\_

(c)  $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

Show all necessary work here:

By the \_\_\_\_\_ test, the series is \_\_\_\_\_

(12) 4. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. (You do not need to show work).

(a)  $\sum_{n=1}^{\infty} \frac{\sin n}{n!}$

(b)  $\sum_{n=1}^{\infty} e^{-n} n!$

(c)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

(16) 5. For the power series  $\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$ , find the following, showing all work.

(a) The radius of convergence  $R$ .

$R =$
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(b) The interval of convergence. (Don't forget to check the end points).

Interval of convergence
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(9) 6. Evaluate the indefinite integral  $\int \frac{t}{1-t^8} dt$  as a power series and give the radius of convergence.

$\int \frac{t}{1-t^8} dt = \sum$	$, R =$
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(9) 7. Find the first three nonzero terms of the Taylor series for  $f(x) = \ln x$  centered at  $a = 2$ .

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