NAME ____________________

STUDENT ID ____________________

RECITATION INSTRUCTOR ____________________

RECITATION TIME ____________________

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DIRECTIONS

1. Write your name, student ID number, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, 4 and 5.
2. The test has five (5) pages, including this one.
3. Write your answers in the boxes provided.
4. You must show sufficient work to justify all answers. Correct answers with inconsistent work may not be given credit.
5. Credit for each problem is given in parentheses in the left hand margin.
6. No books, notes or calculators may be used on this test.

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(10) 1. Evaluate the limit as a number, ∞, or $-\infty$. (You need not show work for this problem).

(a) $\displaystyle\lim_{n\to\infty} \frac{2n^2-4}{-n-5}$

$\boxed{}$

(b) $\displaystyle\lim_{n\to\infty} \cos^{-1}\left(\frac{1}{\sqrt{2}}\,\sin\left(\frac{\pi n}{2n+3}\right)\right)$

$\boxed{}$

(c) $\displaystyle\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$

$\boxed{}$

(12) 2. Circle the letter of the correct response. (You need not show work for this problem).

(a) Which of the following series converge?

(I) $\sum_{n=2}^{\infty} \frac{1-n^2}{1+n}$

(II) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$

(III) $\frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \ldots$

A. (III) only B. none C. (II) only D. (II) and (III) only E. (I) and (II) only

(b) Which of the following statements are true??

(I) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(II) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

(III) If $\lim_{n\to\infty} \sqrt[n]{a_n} = \frac{3}{2}$, then $\sum_{n=1}^{\infty} a_n$ converges

A. (I) only B. (II) only C. (III) only D. none E. (I) and III) only

(10) 3. Let s be the sum of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{1+n+6n^2}$. What is the smallest number of terms you have to add up in order to approximate s with an error less than 0.01? You must show work.

(12) 4. (a) Prove that $\lim_{n\to\infty} n \tan \frac{1}{n} = 1$. (You may use $\lim_{x\to 0} \frac{\sin x}{x} = 1$).

(b) Use (a) to determine whether the series $\sum_{n=1}^{\infty} \frac{\tan \frac{1}{n}}{n}$ is convergent or divergent. You must show all work and name the test you are using, if any.

The series is

(8) 5. Find the sum of the series $\sum_{n=1}^{\infty} 5 \left(\frac{1}{2}\right)^{n+1}$, if it converges.

(8) 6. Determine whether the series $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ diverges, converges conditionally, or converges absolutely. You must justify your answer.

The series

7. Determine whether each series is convergent or divergent. You must show all necessary work and write your conclusion in the small box.

(8) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^n}$

Show all necessary work here:

By the test, the series is

(8) (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

Show all necessary work here:

By the test, the series is

(8) (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$

Show all necessary work here:

By the test, the series is

(16) 8. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\ 2^n}\, x^n$. Don't forget to test for convergence at the end points of the interval. You must show all work.

(10) 9. Given that $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \ldots$, approximate the value of the integral $\int_0^{\frac{1}{10}} e^{-x^2}\,dx$ with error less than 10^{-5}, using the <u>smallest possible</u> number of terms of the series. (You may leave your answer as a sum of fractions).

$$\boxed{\int_0^{\frac{1}{10}} e^{-x^2}\,dx \approx \qquad\qquad}$$