MA 16600 Exam 3, November 2019

Name ____

10–digit PUID number ______________________________________

Recitation Instructor ______________________________________

Recitation Section Number and Time _______________________________

Instructions: **MARK TEST NUMBER 26 ON YOUR SCANTRON**

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 12 problems, each worth 8 points. You will get 4 points for correctly supplying information above and on the scantron.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.

1. The radius of convergence of the series

$$\sum_{k=0}^{\infty} k^k (x+1)^k$$

is

A. 0
B. 1
C. 2
D. 1/2
E. $1/e$

2. Is the series $\displaystyle\sum_{n=0}^{\infty} \sqrt{\frac{1}{2^n} + \frac{3}{4^n}}$ convergent? Why?

A. It is convergent by comparison with $\sum_{n=0}^{\infty} 1/(\sqrt{2})^n$.
B. It is divergent by comparison with $\sum_{n=0}^{\infty} 1/(\sqrt{2})^n$.
C. It is divergent by limit comparison with $\sum_{n=0}^{\infty} 1/(\sqrt{2})^n$.
D. It is convergent by comparison with $\sum_{n=0}^{\infty} \sqrt{3}/2^n$.
E. None of the above answers is correct.

3. The Taylor series of $\ln(e+x)$ about 0 is

A. $1+\sum_{m=1}^{\infty}(-1)^{m+1}\frac{(e+x)^m}{m}$

B. $1+\sum_{m=1}^{\infty}(-1)^{m+1}\frac{x^m}{me^m}$

C. $1+\sum_{m=1}^{\infty}(-1)^{m+1}\frac{x^m}{e^m}$

D. $1+\sum_{m=1}^{\infty}(-1)^{m+1}\frac{x^m}{m!}$

E. $1+\sum_{m=1}^{\infty}(-1)^{m+1}\frac{e^m x^m}{m!}$

4. We use the ratio test to determine if $\sum_{j=1}^{\infty}\frac{4^j(j!)^2}{(2j)!}$ converges. The value of r in the test is

A. 4, hence the series converges.

B. 4, hence the series diverges.

C. 1/4, hence the series converges.

D. 1, hence the series converges.

E. The test is inconclusive.

5. $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1} + (-2)^n}{4^n} =$

A. $14\frac{1}{2}$

B. $9\frac{3}{5}$

C. $12\frac{2}{3}$

D. $11\frac{2}{5}$

E. $10\frac{1}{5}$

6. Which is true for the series I. $\displaystyle\sum_{k=1}^{\infty} \frac{\ln k}{k}$, II. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$?

A. I. converges and II. converges absolutely.

B. I. diverges and II. converges absolutely.

C. Both series converge conditionally.

D. Both series diverge.

E. I. converges conditionally and II. converges absolutely.

7. The first order Taylor polynomial of $4/\sqrt{5-x}$ about $a = 1$ is

A. $2 + \dfrac{x-1}{4}$

B. $4 + \dfrac{x+1}{\sqrt{5}}$

C. $2 - \dfrac{x+1}{\sqrt{5}}$

D. $2 + \dfrac{x-1}{2}$

E. $1 + \dfrac{x-1}{\sqrt{5}}$

8. Which is true for the series $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$?

I. If $a_n \geq 0$ for all n and $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges;

II. If $b_n \geq a_n \geq 0$ for all n and $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ also diverges.

A. Only I.

B. Only II.

C. Both I. and II.

D. Neither is true.

E. None of the above answers is correct.

9. Using Taylor series at 0 to compute $\int_0^{1/2} \cos x^2 \, dx$ we obtain

A. $\sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)2^{k+2}(2k)!}$

B. $\sum_{k=0}^{\infty} \frac{1}{(4k+1)4^k(2k+1)!}$

C. $\sum_{k=0}^{\infty} \frac{2^k}{4(k+1)(2k)!}$

D. $\sum_{k=0}^{\infty} \frac{(-1)^k}{2(4k+1)16^k(2k)!}$

E. $\sum_{k=0}^{\infty} \frac{1}{(2k+1)2^{k+1}(2k)!}$

10. Let $p_3(x)$ denote the 3rd order Taylor polynomial, centered at 0, of e^{-x}. Taylor's Remainder Theorem guarantees that $p_3(1)$ approximates e^{-1} with error $\leq$

A. 1/64

B. 1/48

C. 1/36

D. 1/32

E. 1/24

11. The interval of convergence of the power series $\sum_{p=1}^{\infty} \frac{2^p(x-1)^p}{p}$ is

A. $[1/2, 3/2)$

B. $(-1, 3]$

C. $[1/2, 3]$

D. $(-1, 3)$

E. $(1/2, 3/2)$

12. We approximate $\sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{2i(2i-1)}$ by its n'th partial sum. What is the smallest n for which the Alternating Series Error Estimate guarantees that the error will be $< 10^{-2}$?

A. 3

B. 4

C. 5

D. 6

E. 8