MA 16600
EXAM 2 INSTRUCTIONS
VERSION 01
March 7, 2023

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, 10 of which are worth 8 points and 2 of which are worth 10 points. The maximum possible score is

$$10 \text{ questions } \times 8 \text{ points } + 2 \text{ questions } \times 10 \text{ points } = 100 \text{ points.}$$

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Questions

1. (8 points) Compute $\int \sin^{\frac{5}{2}}(x)\cos^3(x)dx$.

A. $-\frac{2}{7}\sin^{\frac{7}{2}}(x)+\frac{2}{11}\sin^{\frac{11}{2}}(x)+C$

B. $\frac{2}{5}\sin^{\frac{5}{2}}(x)-\frac{2}{9}\sin^{\frac{9}{2}}(x)+C$

C. $-\frac{2}{5}\sin^{\frac{5}{2}}(x)+\frac{2}{9}\sin^{\frac{9}{2}}(x)+C\text{s}$

D. $\frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)+C$

E. $\frac{2}{5}\sin^{\frac{5}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)+C$

2. (8 points) Three students try to compute

$$\int \tan(x)\ dx = \int \frac{\sin(x)}{\cos(x)}\ dx.$$

Elijah: If we use the substitution $u = \sin x$, the integration becomes

$$\int \frac{u}{1-u^2}\ du = \frac{1}{2}\int \left\{\frac{1}{1-u} - \frac{1}{1+u}\right\}\ du.$$

Therefore, the final answer is

$$\frac{1}{2}\{-\ln|1-\sin(x)| - \ln|1+\sin(x)|\} + C.$$

Mia: If we use one more substitution $v = 1 - u^2$ after Elijah's, the integration becomes

$$\int \frac{u}{1-u^2}\ du = -\frac{1}{2}\int \frac{dv}{v}.$$

Therefore, the final answer is $-\dfrac{1}{2}\ln|1-\sin^2(x)| + C$.

Santa: If we use the substitution $w = \cos x$, then the integration becomes

$$-\int \frac{1}{w}dw = -\ln|w| + C.$$

Therefore, the final answer is $-\ln|\cos(x)| + C = \ln|\sec(x)| + C$.

Choose the correct statement about their claims from the following.

A. ONLY Elijah is right.

B. ONLY Mia is right.

C. The strategies by Elijah and Mia are right, but their answers are wrong. They are different from the well-known formula $\int \tan(x)\ dx = \ln|\sec(x)| + C$. Therefore, ONLY Santa is right.

D. ALL three of them are right.

E. Elijah did not include the one more substitution step, which Mia completed. Santa's method is the standard one. So ONLY Mia and Santa are right.

3. (8 points) Compute $\displaystyle\int_0^{2\pi} \sin^2(x)\cos^2(x)\ dx$.

A. $\dfrac{\pi}{8} - \dfrac{1}{16}$

B. $\dfrac{\pi}{8}$

C. $\dfrac{\pi}{4} - \dfrac{1}{16}$

D. $\dfrac{\pi}{4} - \dfrac{1}{32}$

E. $\dfrac{\pi}{4}$

4. (8 points) Compute the integral $\int \tan^4(x)\sec^4(x)\,dx$.

A. $\frac{1}{7}\tan^7(x) - \frac{1}{5}\tan^5(x) + C$

B. $\frac{1}{6}\tan^5(x) - \frac{1}{4}\tan^3(x) + C$

C. $\frac{1}{7}\tan^7(x) + \frac{1}{5}\tan^5(x) + C$

D. $\frac{1}{7}\tan^7(x) - \frac{1}{5}\tan^3(x) + C$

E. $\frac{1}{6}\tan^6(x) - \frac{1}{4}\tan^4(x) + C$

5. (8 points) Compute the integral $\int_0^{\frac{\pi}{4}} \tan(x)\sec^3(x)\,dx$.

A. $\dfrac{4\sqrt{2}-1}{5}$

B. $\dfrac{4\sqrt{2}}{5}$

C. $\dfrac{2\sqrt{2}-1}{3}$

D. $\dfrac{2\sqrt{2}}{3}$

E. $\dfrac{3}{4}$

6. (8 points) We try to compute

$$\int \tan^2(x)\sec(x)\ dx$$

using integration by parts, setting

$$\begin{cases} u &= \tan(x), & v &= \sec(x) \\ du &= \sec^2(x)\ dx, & dv &= \sec(x)\tan(x)\ dx. \end{cases}$$

Then we have

$$\begin{aligned} \int \tan^2(x)\sec(x)dx &= \int udv = uv - \int vdu \\ &= \tan(x)\sec(x) - \int \sec(x)\sec^2(x)\ dx \\ &= \tan(x)\sec(x) - \int \sec(x)\{1+\tan^2(x)\}\ dx \\ &= \tan(x)\sec(x) - \int \sec(x)\ dx - \int \tan^2(x)\sec(x)dx. \end{aligned}$$

Choose the correct statement for the above trial.

A. Since we end up seeing the same formula at the end as we started with, this trial fails to compute the integral.

B. The only substitution we can use is $u = \tan(x)$ or $u = \sec(x)$. For the given integral, neither of them works. The above trial confirms that this integral cannot be computed by any method.

C. This computation gives the formula
$$\int \tan^2(x)\sec(x)\ dx = \tan(x)\sec(x) - \ln|\sec(x)+\tan(x)| + C.$$

D. This computation gives the formula
$$\int \tan^2(x)\sec(x)\ dx = \frac{1}{2}\{\tan(x)\sec(x) - \ln|\sec(x)+\tan(x)|\} + C.$$

E. The above trial fails. We have to use the foumula
$$\int \tan^2(x)\sec(x)\ dx = \int \{\sec^2(x)-1\}\sec(x)\ dx.$$
Then ONLY after we compute $\int \sec^3(x)\ dx$ and $\int \sec(x)\ dx$, we can derive the formula.

7. (10 points) Evaluate the integral

$$\int_0^{\frac{5}{\sqrt{2}}} \frac{x^2}{\sqrt{25-x^2}}\, dx.$$

A. $\frac{25}{8}(\pi - 2)$

B. $\frac{25}{8}\pi$

C. $\frac{25}{4}$

D. 25π

E. $\frac{25}{8}(\pi + 2)$

8. (8 points) Compute the integral

$$\int \frac{\sqrt{x^2-49}}{7x}\,dx \text{ with condition } x > 7.$$

A. $\frac{1}{7}\sqrt{x^2-49} - \sec^{-1}\left(\frac{x}{7}\right) + C$

B. $\sqrt{x^2-7} - \sec^{-1}\left(\frac{x}{7}\right) + C$

C. $7\sqrt{x^2-49} - \sec^{-1}\left(\frac{x}{7}\right) + C$

D. $\sqrt{x^2-7} - 7\sec^{-1}\left(\frac{x}{7})\right) + C$

E. $\frac{1}{7}\sqrt{x^2-49} - \sec^{-1}(x) + C$

9. (8 points) Choose the appropriate trigonometric substitution to compute the integral

$$\int \sqrt{7 + 12x + 4x^2}\, dx.$$

A. $2x + 3 = \sqrt{2} \sin \theta$

B. $2x = \sqrt{7} \tan \theta$

C. $2x + 3 = \sec \theta$

D. $2x + 3 = \sqrt{2} \tan \theta$

E. $2x + 3 = \sqrt{2} \sec \theta$

10. (8 points) Write out the form of the partial fraction decomposition of the function

$$\frac{1}{(x-2)^2(x^2-4)(x^2+4)}.$$

A. $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{(x-2)^3}+\dfrac{D}{x+2}+\dfrac{E}{x^2+4}$

B. $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{(x-2)^3}+\dfrac{D}{x+2}+\dfrac{E}{(x+2)^2}$

C. $\dfrac{Ax+B}{(x-2)^2}+\dfrac{Cx+D}{x^2-4}+\dfrac{Ex+F}{x^2+4}$

D. $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{Cx+D}{x^2-4}+\dfrac{Ex+F}{x^2+4}$

E. $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{(x-2)^3}+\dfrac{D}{x+2}+\dfrac{Ex+F}{x^2+4}$

Note: The letters A, B, C, D, E, F in the partial fractions above represent some appropriate constants.

11. (10 points) Evaluate the integral

$$\int \frac{x^2 + 6x + 17}{x(x^2 + 8x + 17)}\, dx.$$

A. $\ln|x| - 3\tan^{-1}(x+4) + C$

B. $\ln|x| + \tan^{-1}(x) + C$

C. $\ln|x| - 2\tan^{-1}(x+4) + C$

D. $\ln|x| + \ln|x+9| - \ln|x-1| + C$

E. $\ln|x| - 2\ln|x^2 + 8x + 17| + C$

12. (8 points) Evaluate the improper integral $\int_0^\infty xe^{-5x^2}dx$.

A. $\frac{-1}{2}$

B. $\frac{1}{2}$

C. $\frac{-1}{10}$

D. $\frac{1}{10}$

E. The integral is divergent.