NAME _____

10-DIGIT PUID _____

REC. INSTR. _____ REC. TIME _____

LECTURER _____

INSTRUCTIONS:

1. There are 8 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–8.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators or any electronic devices may be used on this exam.
5. Each problem has its own points assigned. The maximum possible score is 100 points.
6. Using a #2 pencil, fill in each of the following items on your scantron sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, put 0 in the first column and then enter the 3-digit section number. For example, for section 016 write 0016. Fill in the little circles.
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–14 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your scantron sheet and your test booklet to your recitation instructor.

(7 pts) 1. Find the formula for the following indefinite integral

$$\int \sin^3 x \cos^2 x \, dx$$

A. $\dfrac{\cos^3 x}{3} + \dfrac{\cos^4 x}{4} + C$

B. $\cos^3 x + \cos^5 x + C$

C. $-\dfrac{\cos^3 x}{3} + \dfrac{\cos^5 x}{5} + C$

D. $\dfrac{\cos^2 x}{2} - \dfrac{\cos^4 x}{4} + C$

E. $-\cos^2 x + \cos^5 x + C$

(7 pts) 2. Evaluate $\displaystyle\int_0^{\pi} \sin^2 x dx$

A. $\dfrac{1}{2}\pi$

B. $\dfrac{1}{3}\pi$

C. 0

D. 1

E. 2

(7 pts) 3. Find the formula for the following indefinite integral

$$\int \frac{\tan^2\theta}{\cos^4\theta}\,d\theta$$

A. $\dfrac{\tan^5 x}{5} + \ln|\sec x| + C$

B. $\dfrac{\tan^5 x}{5} + \dfrac{\tan^3 x}{3} + C$

C. $\dfrac{\sec^5 x}{5} + 3\ln|\sec x| + C$

D. $\dfrac{\sec^5 x}{5} + \dfrac{\sec^3 x}{3} + C$

E. $\dfrac{\sec^5 x}{5} - \dfrac{\sec^3 x}{3} + C$

(7 pts) 4. Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \tan^3\theta\sec\theta\,d\theta$

A. $\dfrac{2-\sqrt{2}}{3}$

B. $\dfrac{2-\sqrt{2}}{3} + C$

C. $\dfrac{1}{3}$

D. $\dfrac{1}{\sqrt{2}}$

E. $2-\sqrt{2}$

(7 pts) 5. Find the formula for the following indefinite integral:

$$\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$$

A. $\sqrt{x^2-9}\,\ln|x|+C$

B. $\sqrt{x^2-9}+\ln|x|+C$

C. $\sqrt{x^2-9}+C$

D. $\dfrac{\sqrt{x^2-9}}{9x}+C$

E. $\dfrac{\sqrt{x^2-9}}{18x}+C$

(7 pts) 6. Find the formula for the following indefinite integral:

$$\int \frac{1}{\sqrt{x^2-6x+13}}\,dx$$

A. $\ln|\sqrt{x^2-6x+13}+x-3|+C$

B. $\ln|\sqrt{x^2-6x+13}|+C$

C. $\ln|\sqrt{x^2-6x+13}+x|+C$

D. $\sin^{-1}\left(x^2-6x+13\right)+C$

E. $\tan^{-1}\left(x^2-6x+13\right)+C$

(7 pts) 7. Find the formula for the following indefinite integral

$$\int \frac{x^2}{x^2-4}\,dx =$$

A. $x - \ln\left|\dfrac{x-2}{x+2}\right| + C$

B. $1 + \ln\left|\dfrac{x-2}{x+2}\right| + C$

C. $x + \ln\left|\dfrac{x-2}{x+2}\right| + C$

D. $1 + \ln|x^2-4| + C$

E. $x + 2(x^2-4)^{-2} + C$

(7 pts) 8. Find the formula for the following indefinite integral

$$\int \frac{10}{(x-1)(x^2+9)}\,dx$$

A. $5\ln|x-1| + \dfrac{5}{3}\tan^{-1}\dfrac{x}{3} + C$

B. $\ln|x-1| - \dfrac{1}{2}\ln(x^2+9) - \dfrac{1}{3}\tan^{-1}\dfrac{x}{3} + C$

C. $10\ln|x-1| + \dfrac{10}{3}\tan^{-1}\dfrac{x}{3} + C$

D. $2\ln|x-1| + \dfrac{1}{3}\tan^{-1}\dfrac{x}{3} + C$

E. $\ln|x-1| + \ln(x^2+9) + \dfrac{1}{3}\tan^{-1}\dfrac{x}{3} + C$

(7 pts) 9. Evaluate $\displaystyle\int_9^{16} \frac{\sqrt{x}}{x-4}\,dx =$

A. $\dfrac{4}{3}$

B. $1 + \ln\dfrac{4}{3}$

C. $2\left(3 - \ln\dfrac{3}{5}\right)$

D. 3

E. $2\left(1 + \ln\dfrac{5}{3}\right)$

(7 pts) 10. If we use Simpson's rule to approximate the integral

$$\int_0^{\pi} x^2 \sin x \; dx$$

with

$$f(x) = x^2 \sin x \text{ and } n = 4,$$

then the formula is given by

A. $\dfrac{\pi}{4}\{f(0) + 4f(\pi/4) + 2f(\pi/2) + 4f(3\pi/4) + f(\pi)\}$

B. $\dfrac{\pi}{8}\{f(0) + 4f(\pi/4) + 2f(\pi/2) + 4f(3\pi/4) + f(\pi)\}$

C. $\dfrac{\pi}{12}\{f(0) + 4f(\pi/4) + 2f(\pi/2) + 4f(3\pi/4) + f(\pi)\}$

D. $\dfrac{\pi}{8}\{f(0) + 2f(\pi/4) + 2f(\pi/2) + 2f(3\pi/4) + f(\pi)\}$

E. $\dfrac{\pi}{4}\{f(\pi/8) + f(3\pi/8) + f(5\pi/8) + f(7\pi/8)\}$

(7 pts) 11. Which of the following integrals are divergent?

(i) $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}\,dx$

(ii) $\displaystyle\int_0^4 \frac{\ln x}{x}dx$

(iii) $\displaystyle\int_2^3 \frac{1}{\sqrt{3-x}}\,dx$

A. (i) only

B. (ii) only

C. (ii) and (iii)

D. None

E. All

(8 pts) 12. Find the length of the curve $y = \ln(\cos x)$, $0 \le x \le \pi/3$.

A. $\ln 3 - 1$

B. $\ln 3 - \ln 2$

C. $\ln(2+\sqrt{3})$

D. $\ln(2\sqrt{3}+1)$

E. $\ln\sqrt{3} + \ln 2$

(7 pts) 13. Set up but do not evaluate the integral for the area of the surface obtained by rotating the curve $y = \tan^{-1} x$, $0 \le x \le 1$ about the y-axis.

A. $\displaystyle\int_0^1 2\pi\sqrt{1+\frac{1}{(1+x^2)^2}}\,dx$

B. $\displaystyle\int_0^1 2\pi \tan^{-1} x\sqrt{1+\frac{1}{(1+x^2)^2}}\,dx$

C. $\displaystyle\int_0^1 2\pi y\sqrt{1+\sec^2 y}\,dy$

D. $\displaystyle\int_0^1 2\pi x\sqrt{1+\frac{1}{(1+x^2)^2}}\,dx$

E. $\displaystyle\int_0^1 \pi x^2\sqrt{1+\frac{1}{(1+x^2)^2}}\,dx$

(8 pts) 14. Let $(\overline{x}, \overline{y})$ be the centroid of the region bounded by the curves

$y = \sin x, \quad y = \cos x, \quad 0 \le x \le \pi/4.$

Find $\overline{y}$.

A. $\dfrac{4\pi\sqrt{2}-1}{\sqrt{2}-1}$

B. $\sqrt{2}-1$

C. $\dfrac{1}{4}\pi\sqrt{2}-1$

D. $\dfrac{1}{4(\sqrt{2}-1)}$

E. $\dfrac{1}{\sqrt{2}-1}$