

MA 16600  
EXAM 1 INSTRUCTIONS  
VERSION 01  
February 7, 2023

Your name \_\_\_\_\_ Your TA's name \_\_\_\_\_

Student ID # \_\_\_\_\_ Section # and recitation time \_\_\_\_\_

1. You must use a #2 pencil on the scantron sheet (answer sheet).
2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.
3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.
4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.
5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.
6. **Sign the scantron sheet.**
7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.
8. There are 12 questions, 10 of which are worth 8 points and 2 of which are worth 10 points. The maximum possible score is  
 $10 \text{ questions} \times 8 \text{ points} + 2 \text{ questions} \times 10 \text{ points} = 100 \text{ points}.$
9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.
10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.
11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

## Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

## Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: \_\_\_\_\_

STUDENT SIGNATURE: \_\_\_\_\_

## Questions

1. (8 points) Find the center and radius of the sphere defined by the following equation:

$$x^2 - 8x + y^2 - 4y + z^2 + 6z = 7$$

- A. Center  $(4, 2, -3)$  and Radius 6
- B. Center  $(4, 2, -3)$  and Radius  $\sqrt{7}$
- C. Center  $(-4, -2, 3)$  and Radius 6
- D. Center  $(-4, -2, 3)$  and Radius  $\sqrt{7}$
- E. Center  $(4, 2, -3)$  and Radius 7

2. (8 points) Find a vector

- (a) that has the opposite direction to the vector  $\overrightarrow{PQ}$ , and
- (b) that has length 5.

We are given that

$$\begin{cases} P = (-1, 3, 2) \\ Q = (2, -2, 6). \end{cases}$$

- A.  $\frac{1}{\sqrt{2}}\langle 3, -5, 4 \rangle$
- B.  $\frac{-1}{\sqrt{50}}\langle 3, -5, 4 \rangle$
- C.  $\frac{-1}{\sqrt{2}}\langle 3, -5, 4 \rangle$
- D.  $\frac{5}{\sqrt{66}}\langle 1, 1, 8 \rangle$
- E.  $\frac{1}{\sqrt{66}}\langle 1, 1, 8 \rangle$

3. (8 points) Find an equation of the plane

$$\begin{cases} \text{passing through the point } P(2, 1, 5), \text{ and} \\ \text{perpendicular to the planes } 3x + y - 5z = 2 \text{ and } -x + 2y + z = 7. \end{cases}$$

A.  $2x + y + 5z = 3 \cdot (-1) + 1 \cdot 2 + (-5) \cdot 1$

B.  $11x + 2y + 7z = 0$

C.  $11(x - 2) + 2(y - 1) + 7(z - 5) = 0$

D.  $11(x - 2) - 2(y - 1) + 7(z - 5) = 0$

E.  $(3 \cdot (-1))x + (1 \cdot 2)y + ((-5) \cdot 1)z = 2 + 1 + 5$

4. (8 points) Find  $\text{Proj}_{\mathbf{v}}\mathbf{u}$  (the orthogonal projection of  $\mathbf{u}$  onto  $\mathbf{v}$ ), where  $\mathbf{u} = \langle 1, -3, 2 \rangle$  and  $\mathbf{v} = \langle 3, 1, 5 \rangle$ .

A.  $\frac{10}{35}\mathbf{v}$

B.  $\frac{10}{35}\mathbf{u}$

C.  $\frac{10}{\sqrt{35}}\mathbf{v}$

D.  $\frac{10}{\sqrt{35}}\mathbf{u}$

E.  $\frac{1}{35}\mathbf{v}$

5. (8 points) We have the following 4 points A,B,C,D in the 3-space:

$$\begin{cases} A = (1, -2, 3) \\ B = (4, -1, 3) \\ C = (5, -2, 4) \\ D = (6, -3, 5) \end{cases}$$

Set

$$\begin{cases} \vec{u} = \overrightarrow{AB} \\ \vec{v} = \overrightarrow{AC} \\ \vec{w} = \overrightarrow{AD} \end{cases}$$

Choose the correct statement from below:

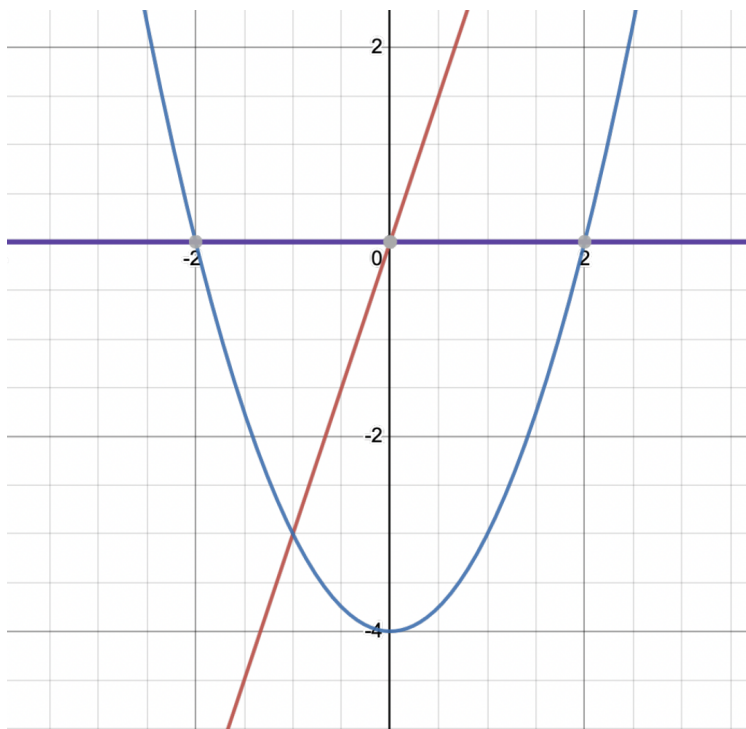
- A.  $|\vec{u} \cdot (\vec{v} \times \vec{w})| = 6$  and hence 4 points A,B,C,D are NOT coplanar.
- B.  $|\vec{u} \cdot (\vec{v} \times \vec{w})| = -6$  and hence 4 points A,B,C,D are NOT coplanar.
- C.  $|\vec{u} \cdot (\vec{v} \times \vec{w})| = 3$  and hence 4 points A,B,C,D are NOT coplanar.
- D.  $|\vec{u} \cdot (\vec{v} \times \vec{w})| = 0$  and hence 4 points A,B,C,D are coplanar.
- E.  $|\vec{u} \cdot (\vec{v} \times \vec{w})| = 0$  and hence 4 points A,B,C,D are NOT coplanar.

6. (8 points) Consider the region **in the 3rd quadrant** bounded by the curves  $y = 3x$ ,  $y = x^2 - 4$  and  $y = 0$ .

(See the picture below.)

Choose the right formulas to compute the area of the region

- (i) in terms of  $x$ , and  
(ii) in terms of  $y$ .



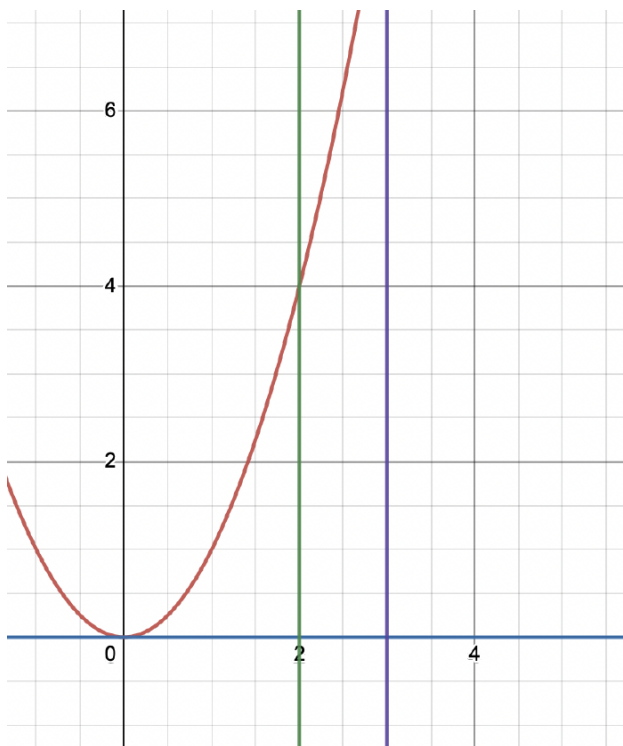
- A. (i)  $\int_{-4}^0 \{(x^2 - 4) - 3x\} dx$ , (ii)  $\int_{-3}^0 \left\{ \sqrt{y+4} - \frac{1}{3}y \right\} dy$   
B. (i)  $\int_{-4}^0 \{3x - (x^2 - 4)\} dx$ , (ii)  $\int_{-3}^0 \left\{ \frac{1}{3}y - \sqrt{y+4} \right\} dy$   
C. (i)  $\int_{-2}^{-1} (-x^2 + 4) dx + \int_{-1}^0 (-3x) dx$  (ii)  $\int_{-3}^0 \left\{ \frac{1}{3}y + \sqrt{y+4} \right\} dy$   
D. (i)  $\int_{-2}^{-1} (-x^2 + 4) dx + \int_{-1}^0 (-3x) dx$  (ii)  $\int_{-3}^0 \left\{ \frac{1}{3}y - \sqrt{y+4} \right\} dy$   
E. (i)  $\int_{-2}^{-1} (x^2 - 4) dx + \int_{-1}^0 (3x) dx$  (ii)  $\int_{-3}^0 \left\{ \frac{1}{3}y + \sqrt{y+4} \right\} dy$



7. (8 points) Choose the right formulas to compute the volume of the solid obtained from rotating **about the line**  $x = 3$  the region given below, using

- (i) Washer method,
- (ii) Shell method.

The region is bounded by  $y = x^2$ ,  $x = 2$  and  $y = 0$ .



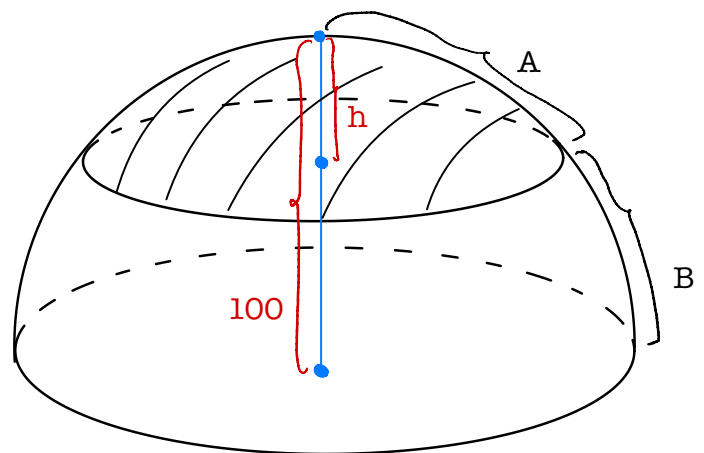
- A. (i)  $\int_0^4 \pi(\sqrt{y})^2 dy$ , (ii)  $\int_0^2 2\pi x \cdot x^2 dx$
- B. (i)  $\int_0^2 2\pi x \cdot x^2 dx$ , (ii)  $\int_0^4 \pi \{(\sqrt{y})^2 - 2^2\} dy$
- C. (i)  $\int_0^4 \pi \{(3 - \sqrt{y})^2 - 1^2\} dy$ , (ii)  $\int_0^2 2\pi(3 - x)x^2 dx$
- D. (i)  $\int_0^4 \pi \{(3 + \sqrt{y})^2 - 1^2\} dy$ , (ii)  $\int_0^2 2\pi(x - 3)x^2 dx$
- E. (i)  $\int_{-2}^{-1} (x^2 - 3) dx + \int_{-1}^0 (3x) dx$  (ii)  $\int_{-3}^0 \left(\frac{1}{3}y + \sqrt{y + 4}\right) dy$

8. (8 points) There is a small planet  $X$ , which has the shape of a sphere of radius 100 km. Countries  $A$  and  $B$  occupy the northern hemisphere, with country  $A$  ruling the northern cap of height  $h$ , including the north pole, and country  $B$  ruling the rest. (See the picture below.)

The areas of these two countries are the same.

What is the height  $h$  ?

- A. 25 km
- B.  $50(2 - \sqrt{2})$  km  $\approx 29.29$  km
- C. 50 km
- D. 75 km
- E.  $50\sqrt{2}$  km  $\approx 70.71$  km



9. (8 points) Using “Integration by Parts”, compute and choose the right answer for the indefinite integral  $\int x^{2023} \ln x \, dx$ .

A.  $\frac{x^{2024} \ln x}{2024} - \frac{x^{2025}}{(2024)(2025)} + C$

B.  $\frac{x^{2024} \ln x}{2024} - \frac{x^{2024}}{(2024)^2} + C$

C.  $x^{2024} \ln x - x^{2024} + C$

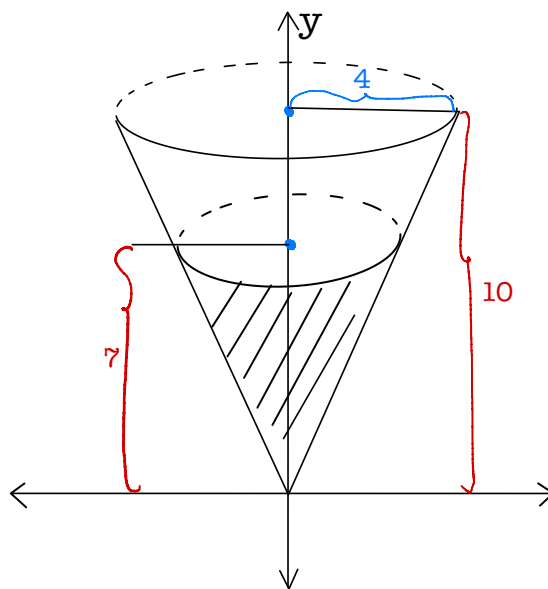
D.  $x^{2024} \ln x - \frac{x^{2024}}{2024} + C$

E.  $x^{2024} \ln x - \frac{x^{2024}}{(2024)^2} + C$

10. (8 points) Find the right formula for the work required to empty the water from the reversed conical tank, where the radius of the top is 4 m and the height is 10 m. The original depth of the water is 7 m and you are only allowed to throw away the water from the top of the tank. Use  $\rho$  kg/m<sup>3</sup> for the density of the water, and  $g$  m/sec<sup>2</sup> for the acceleration constant by the gravity.

Choose the coordinate system so that the vertex at the bottom is the origin of the  $y$ -axis.

(See the picture.)

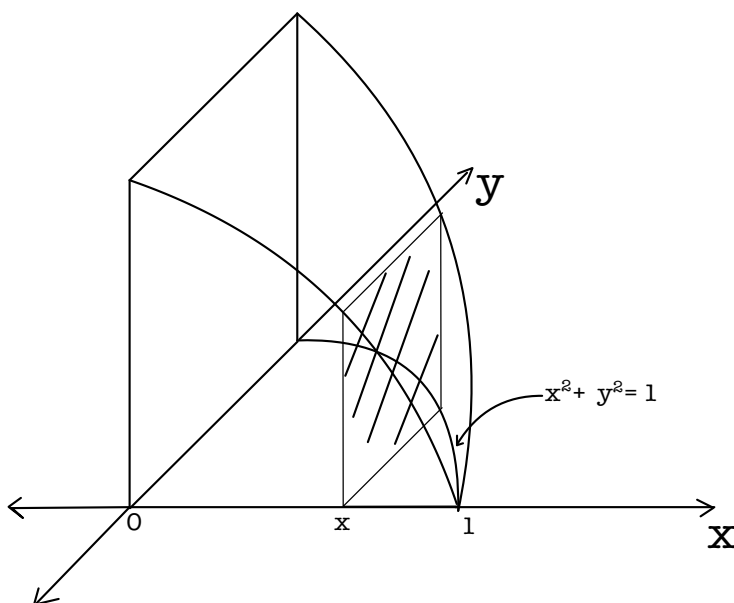


- A.  $\int_7^{10} \rho g \pi \left( \frac{4}{10} y \right)^2 (10 - y) dy$
- B.  $\int_0^{10} \rho g \pi \left( \frac{4}{10} (10 - y) \right)^2 y dy$
- C.  $\int_0^3 \rho g \pi \left( \frac{4}{10} y \right)^2 y dy$
- D.  $\int_0^7 \rho g \pi \left( \frac{4}{10} y \right)^2 (10 - y) dy$
- E.  $\int_0^{10} \rho g \pi \left( \frac{4}{10} y \right)^2 y dy$

11. (10 points) Find the volume of the solid  $S$  described below:

- (a) The base of  $S$  is in the 1st quadrant and a quarter of a circular disk of radius 1 with the origin being the center.
- (b) The cross section perpendicular to the base and  $x$ -axis are squares with one side on the base.

(See the picture.)



- A.  $\frac{4}{15}$
- B.  $\frac{8}{5}$
- C.  $\frac{2}{3}$
- D.  $\frac{1}{3}$
- E.  $\frac{\pi}{3}$

12. (10 points) Find the length of the curve

$$y = f(x) = \frac{e^x + e^{-x}}{2}$$

on the interval  $[0, \ln 5]$ .

- A.  $\sqrt{\frac{24}{5}}$
- B.  $\ln 5 + \frac{12}{5}$
- C.  $\ln 3 + \frac{1}{5}$
- D.  $\frac{12}{5}$
- E.  $\frac{24}{5}$