MA 16500

FINAL EXAM INSTRUCTIONS

VERSION 01

December 12, 2018

Your name ________________ Your TA's name ________________

Student ID # ____________ Section # and recitation time ________

**1.** You must use a #2 pencil on the scantron sheet (answer sheet).

**2.** Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

**3.** On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

**4.** Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

**5.** Fill in the four-digit **SECTION NUMBER**.

**6.** Sign the scantron sheet.

**7.** Blacken your choice of the correct answer in the space provided for each of the questions 1–25. **All the answers must be marked on the scantron sheet.** In case what is marked on the scantron sheet is different from what is marked on the exam booklet, we compute the final score based upon what is marked on the scantron sheet.

**8.** While marking all your answers on the scantron sheet, you should **show your work on the exam booklet**. In case of a suspicious activity of academic dishonesty and/or under certain circumstances, we require that the correct answer on the scantron sheet must be supported by the work on the exam booklet.

**9.** There are 25 questions, each worth 8 points. The maximum possible score is $8 \times 25 = 200$ points.

**10.** **NO calculators, electronic device, books, or notes are allowed.** Use the back of the test pages for scrap paper.

**11.** After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

**12.** If you finish the exam before 5:25 PM, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 5:25 PM, you should REMAIN SEATED until your TA/Proctor comes and collects your scantron sheet and exam booklet. Do NOT form a line in front of your TA while waiting to turn in the scantron sheet and exam booklet. Leaving your seat before submitting the scantron sheet and exam booklet might be regarded as an action of academic dishonesty.

# Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

# Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____________________________________________

STUDENT SIGNATURE: ________________________________________

## Questions

**1.** Find the domain of the function

$$f(x) = \sqrt{\frac{1}{2 - \ln(x-1)}}.$$

A. $(0,\ e^2)$

B. $(1,\ e^2+1)$

C. $(-\infty,\ 1] \cup [e^2+1,\ \infty)$

D. $(1,\ e)$

E. $(0,\ 1) \cup (1,\ e) \cup [e+1,\ \infty)$

**2.** Starting with the graph of $y = e^{-3x}$, write down the equation of the graph that results from reflecting about the line $x = 1$.

A. $-e^{-3x+6}$

B. $e^{3x-2}$

C. $-e^{-3x-6}$

D. $-e^{-3x+2}$

E. $e^{3x-6}$

**3.** Consider the function $f(x) = 1 - \sqrt{x+1}$ over the domain $x \geq -1$.

Choose the correct statement from below.

A. $f$ is invertible and $f^{-1}(x) = \dfrac{1}{1-\sqrt{x+1}}$ with the domain $-1 \leq x < 0$.

B. $f$ is invertible and $f^{-1}(x) = \dfrac{-x}{1+\sqrt{x+1}}$ with the domain $x \geq -1$.

C. $f$ is invertible and $f^{-1}(x) = x^2 - 2x$ with the domain $x \leq 1$.

D. $f$ is invertible and $f^{-1}(x) = x^2 - 2x$ with the domain $x \geq 1$.

E. $f$ is not invertible.

**4.** If we use the linear approximation for $f(x) = \sqrt[4]{x}$ at $a = 16$,

then which of the following best describes the estimate for $\sqrt[4]{16.16}$ ?

A. 2.001

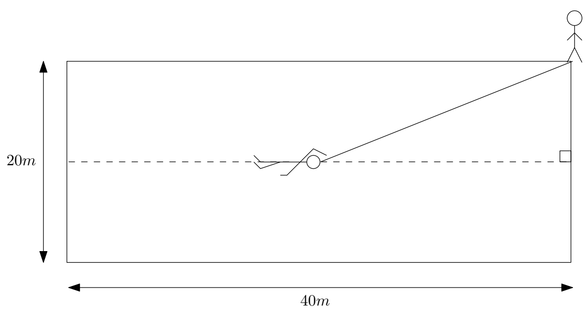
B. 2.005

C. 2.02

D. 2.05

E. 2.2

**5.** Fred is swimming a race in a pool 40 m long and 20 m wide. He is swimming his race in the center lane so that he is always 10m from the either side. His coach is standing at the corner of the pool by the finish line. If Fred swims at a constant rate of 2 m/s, what is the rate at which the distance between Fred and his coach is decreasing when he is halfway through the race.




A. $\dfrac{\sqrt{5}}{4}$ m/s

B. $\sqrt{5}$ m/s

C. $\dfrac{\sqrt{5}}{2}$ m/s

D. $\dfrac{2}{\sqrt{5}}$ m/s

E. $\dfrac{4}{\sqrt{5}}$ m/s

**6.** Suppose that the base of a triangle increases at a rate of 2 in/min and the area increases at a rate of 24 in$^2$/min. At what rate is the altitude of the triangle changing when the base is 4 in and the area is 16 in$^2$?

A. 2 in/min

B. 4 in/min

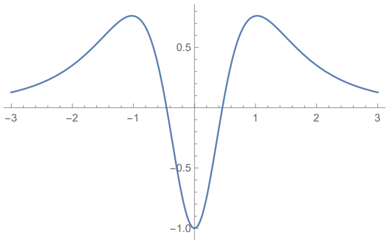
C. 6 in/min

D. 8 in/min

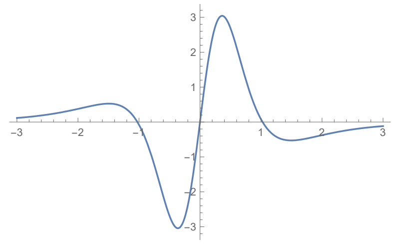
E. 10 in/min

**7.** The graphs of $y = f'(x)$ and $y = f''(x)$ are given below.

$y = f'(x)$

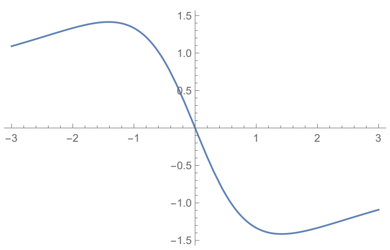



$y = f''(x)$

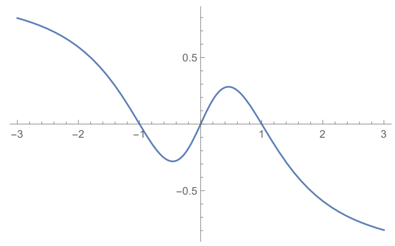



Which of the following then can be the graph of $y = f(x)$.

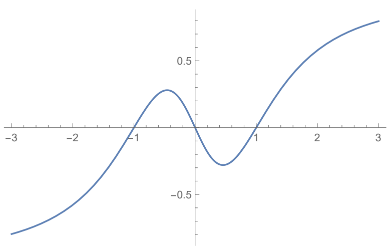
A.



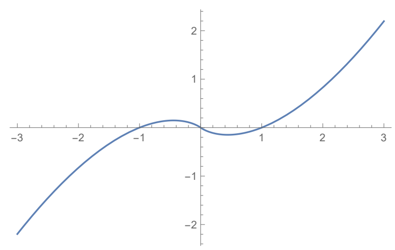

B.



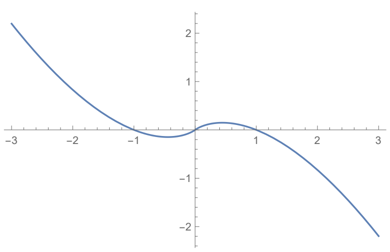

C.



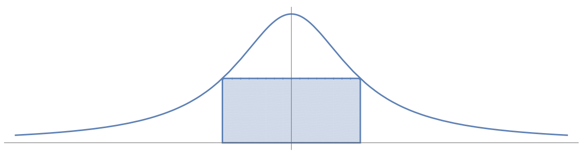

D.




E.



**8.** What is the area of the largest rectangle that can fit in the region above the $x$-axis and below the curve $y = \dfrac{1}{1+4x^2}$ ?



A. $1/2$

B. $2/5$

C. $1/\sqrt{2}$

D. $1$

E. $1/4$

**9.** Compute the following limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1+\frac{3i}{n}} \cdot \frac{3}{n}$$

HINT: Consider what integration the limit represents.

A. $\dfrac{2}{3}$

B. $\dfrac{14}{3}$

C. $\dfrac{1}{3}$

D. $\dfrac{14}{9}$

E. $\dfrac{7}{3}$

**10.** A colony of bacteria is growing and reproducing in a way so that the population doubles every 5 hours. If there are currently 100 individual bacteria in the colony, what is a formula for the number of bacteria in the colony $t$ hours from now?

A. $P(t) = 100e^{\frac{\ln(2)}{5}t}$.

B. $P(t) = 100e^{5\ln(2)t}$.

C. $P(t) = 100e^{-\frac{\ln(2)}{5}t}$.

D. $P(t) = 100e^{-5\ln(2)t}$.

E. $P(t) = 100e^{\ln(2/5)t}$.

**11.** Find the values of $a$ and $b$ so that the function

$$f(x) = \begin{cases} \dfrac{x^2 - x}{x - 1} & \text{if } \; x < 1 \\ ax^2 + 2x + b & \text{if } \; 1 \leq x < 2 \\ x + a & \text{if } \; 2 \leq x. \end{cases}$$

is continuous on $(-\infty, \infty)$.

A. $a = \frac{1}{2}, \; b = -2$

B. $a = -\frac{1}{2}, \; b = -\frac{1}{2}$

C. $a = 2, \; b = -1$

D. $a = \frac{3}{2}, \; b = -\frac{1}{4}$

E. No matter how we choose the values for $a$ and $b$, the function $f(x)$ can never be continuous on $(-\infty, \infty)$.

**12.** Which of the following line is parallel to the tangent line to $\sqrt{y} + \sqrt{x} = 5$ at $(9, 4)$?

A. $y = -\dfrac{2}{3}x - 2$

B. $y = x - \dfrac{8}{27}$

C. $y = \dfrac{1}{3}x + 2$

D. $y = -2x - 3$

E. $y = \dfrac{1}{2}x + 3$

**13.** The position $s$ of a particle is given as a function of time $t \geq 0$ by the formula

$$s = f(t) = \frac{1}{3}t^3 - 5t^2 + 16t.$$

Identify ALL time intervals in $(0, \infty)$ when the particle is **speeding up**.

A. $(2, 8)$

B. $(2, 5)$ and $(8, \infty)$

C. $(0, 2)$ and $(8, \infty)$

D. $(5, \infty)$

E. $(8, \infty)$

**14.** Compute the derivative of $f(x) = x^{\ln(x)}$.

A. $f'(x) = \ln(x)x^{\ln(x)-1}$

B. $f'(x) = (\ln(x)+1)x^{\ln(x)}$

C. $f'(x) = (2\ln x)x^{\ln(x)-1}$

D. $f'(x) = \ln(x)x^{\ln(x)}$

E. $f'(x) = \dfrac{2}{\ln(x)}x^{\ln(x)}$

**15.** Compute the following definite integral

$$\int_{\pi/4}^{\pi/2} \cot(x)\, dx.$$

A. $-1$

B. $1$

C. $\dfrac{1}{2}\ln(2)$

D. $\ln(2)$

E. $\dfrac{1}{\sqrt{2}}$

**16.** Let $G(x) = \displaystyle\int_0^{x^2} \{\sin(2t) + 2\sec(2t)\}\, dt$.

What is $G'(0)$?

WARNING: The upper integration limit in "$\displaystyle\int_0^{x^2} \{\sin(2t) + 2\sec(2t)\}\, dt$"

is $x^2$ and not $x$.

A. 0

B. 2

C. 4

D. $\dfrac{1}{2}$

E. $\dfrac{9}{4}$

**17.** Compute the following limits:

(a) $\lim_{x\to 0^+} (\sin x)^{2\tan x}$

(b) $\lim_{x\to 0} \dfrac{\cos x - 1}{x^2}$

A. (a) $\infty$ (b) DNE

B. (a) 1 (b) $-\dfrac{1}{2}$

C. (a) 0 (b) 2

D. (a) 0 (b) $\dfrac{1}{2}$

E. (a) 1 (b) 0

**18.** The derivative of the function f is given by

$$f'(x) = (x-4)^5(x-2)^2x^3(x+2)^7(x+4)^4$$

Let $M$ be the number of values of $x$ at which local maximum is achieved, and $m$ is the number of values of $x$ at which local minimum is achieved. What is $M - m$?

A. $-2$

B. $-1$

C. $0$

D. $1$

E. $2$

**19.** If $\sin(\theta) = \dfrac{-1}{3}$ and $\pi < \theta < \dfrac{3\pi}{2}$, then $\cos(\theta) = ?$

A. $\dfrac{-2\sqrt{2}}{3}$

B. $\dfrac{2\sqrt{2}}{3}$

C. $\dfrac{-\sqrt{2}}{3}$

D. $\dfrac{-\sqrt{3}}{2}$

E. $\dfrac{\sqrt{3}}{2}$

**20.** Compute the following limits:

(a) $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

(b) $\lim_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{\sin x}\right)$

A. (a) 1/2 (b) 0

B. (a) 0 (b) 0

C. (a) 0 (b) DNE

D. (a) 2 (b) DNE

E. (a) 1/2 (b) $\infty$

**21.** Compute the absolute maximum and absolute minimum of the function $f(x) = 2x^3 + 3x^2 - 12x - 9$ on the interval $[-3, 3]$.

A. Max = 36, Min = 0

B. Max = 36, Min = $-16$

C. Max = 11, Min = $-16$

D. Max = 11, Min = 0

E. Max = 36, Min = $-9$

**22.** Evaluate the integration

$$\int_0^{\sqrt{3}} x^3 \sqrt{1+x^2}\, dx.$$

A. $\dfrac{2}{15}\left(1+\sqrt{2}\right)$

B. $\dfrac{4}{15}\left(1+\sqrt{2}\right)$

C. $\dfrac{\pi}{12}$

D. $\dfrac{13}{15}$

E. $\dfrac{58}{15}$

**23.** Find the algebraic expression for $\tan(2\sin^{-1}(x))$.

HINT:

- $\tan(2\theta) = \dfrac{\sin(2\theta)}{\cos(2\theta)}$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = 1 - 2\sin^2\theta$

A. $\dfrac{x}{\sqrt{1-x^2}}$

B. $\dfrac{1}{\sqrt{1-x^2}}$

C. $\dfrac{2x}{1-2x^2}$

D. $\dfrac{2x\sqrt{1-x^2}}{1-2x^2}$

E. $\dfrac{1-x^2}{1+x^2}$

**24.** Which of the following is a focus of the ellipse given by the equation

$$\frac{(x-1)^2}{25} + \frac{(y+4)^2}{9} = 1?$$

A. $(6, -3)$

B. $(-4, 2)$

C. $(3, -1)$

D. $(5, -4)$

E. $(-5, -2)$

**25.** Find an equation of the hyperbola with foci $(\pm 4\sqrt{5}, 0)$ and asymptotes $y = \pm 2x$.

A. $-4x^2 + y^2 = 4$

B. $4x^2 - y^2 = 64$

C. $-4x^2 + y^2 = 64$

D. $4x^2 - y^2 = 80$

E. $4x^2 - y^2 = 4$