MA 16500
FINAL EXAM INSTRUCTIONS
VERSION 01
DECEMBER 16, 2014

Your name ____________________ Your TA's name ____________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–25. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 25 questions, each worth 8 points. The maximum possible score is

   $8 \times 25 = 200$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 12:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 12:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.

2. Students may not open the exam until instructed to do so.

3. No student may leave in the first 20 min or in the last 5 min of the exam.

4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.

5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.

6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.

2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.

3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.

4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____________________________________

Questions

1. Find the domain of definition for the function $f(x) = \ln(x^2 - 6x + 8)$.

A. $(2, 4)$

B. $(-\infty, 4)$

C. $(4, \infty)$

D. $(2, 4) \cup (2, \infty)$

E. $(-\infty, 2) \cup (4, \infty)$

2. Suppose $f(x)$ and $g(x)$ are differentiable functions with $f'(3) = 5$ and $g'(3) = 2$. Evaluate the limit

$$\lim_{x \to 3} \frac{f(x) - f(3)}{g(x) - g(3)}.$$

A. $\frac{2}{5}$

B. $\frac{5}{2}$

C. ∞

D. 0

E. The limit does not exist.

3. Evaluate

$$\lim_{x\to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4}.$$

A. $\frac{1}{12}$

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. $\frac{1}{24}$

E. The limit does not exist.

4. Which of the following is the derivative of $e^{2x+\sin(3x^3)}$ at $x = 0$?

A. 3

B. 2

C. $\frac{3}{4}e$

D. $\frac{2}{9}$

E. $\frac{9}{4}$

5. Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} 3a \cdot \frac{\sin(2x)}{x} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ 4b\cos(2x) & \text{if } x > 0 \end{cases}$$

Find the values of a and b such that the function is continuous over $(-\infty, \infty)$.

A. $a = \frac{1}{3}$, $b = 1$
B. $a = \frac{1}{6}$, $b = 2$
C. $a = \frac{2}{3}$, $b = 1$
D. $a = \frac{1}{3}$, $b = \frac{1}{2}$
E. $a = \frac{1}{6}$, $b = 2$

6. Find a formula for the inverse of the function

$$f(x) = \frac{3x+2}{5x+6}.$$

A. $f^{-1}(x) = \frac{2x+6}{3x-2}$
B. $f^{-1}(x) = \frac{5x-2}{x-6}$
C. $f^{-1}(x) = \frac{-6x+1}{3x-2}$
D. $f^{-1}(x) = \frac{6x-2}{-5x+3}$
E. $f^{-1}(x) = -\frac{5x+6}{3x+2}$

7. Find the equation of the tangent line to the curve defined by the equation

$$\sin(xy) - 2y = -\pi x^2 + 1$$

at the point $(x, y) = (1, \frac{\pi}{2})$.

A. $y - \pi x + \frac{\pi}{2} = 0$

B. $2y - \pi x + \frac{\pi}{2} = 0$

C. $4y - 2\pi x - 2\pi = 0$

D. $2y + \pi x - 2\pi = 0$

E. $y - \pi x - \frac{\pi}{2} = 0$

8. Consider the function $y = \ln|\sec 5x + \tan 5x|$.
Compute its derivative $\frac{dy}{dx}$.

A. $\sec 5x$

B. $|\sec 5x|$

C. $5 \sec 5x$

D. $5|\sec 5x|$

E. $\frac{5 \sec 5x \tan 5x + 25 \sec^2 5x}{\sec 5x + \tan 5x}$

9. Compute

$$\lim_{x\to\infty} \frac{2x^2+\sqrt{x^4-3x}}{3x+2\sqrt{x^4+5x^2}}$$

A. 3

B. $\frac{3}{5}$

C. $\frac{3}{2}$

D. $\frac{1}{2}$

E. ∞

10. Use the linear approximation of the function $f(x) = \ln x$ at $a = e$ to estimate the number $\ln 2.9$.

A. 0.1

B. $2.9 - e$

C. $\frac{2.9}{e}$

D. 0.083

E. $\frac{2.9-e}{e}$

11. Find the absolute maximum a and absolute minimum b of the function

$$f(x) = \sin x + \cos x$$

on the closed interval $[0, \pi/3]$

A. $a = \sqrt{2},\ b = \frac{1+\sqrt{3}}{2}$

B. $a = \sqrt{2},\ b = 1$

C. $a = \frac{1+\sqrt{3}}{2},\ b = 1$

D. $a = \frac{1+\sqrt{3}}{2},\ b = \sqrt{2}$

E. $a = 1,\ b = \frac{\sqrt{2}}{2}$

12. Find the interval where the function $f(x) = x^3 - 2x + 1$ takes the value 4, using the Intermediate Value Theorem.

A. $(-3, -2)$

B. $(-2, -1)$

C. $(-1, 0)$

D. $(0, 1)$

E. $(1, 2)$

13. Evaluate

$$\lim_{x \to 2^+} \left(\frac{x}{2}\right)^{\frac{3}{x-2}}$$

A. e^3

B. e

C. $e^{\frac{3}{2}}$

D. 1

E. ∞

14. We have a differentiable function $y = f(x)$ over $(-\infty, \infty)$ such that

(a) $f'(1) = 0$, and

(b) $f''(x) = x^2(x+1)^3(x-1)^4$.

Choose the right statement from below.

A. $f(1)$ is a local maximum of the function $y = f(x)$.

B. $f(1)$ is a local minimum of the function $y = f(x)$.

C. $f(1)$ is neither a local maximum nor a local minimum of the function $y = f(x)$.

D. We cannot tell if $f(1)$ is a local maximum or local minimum of the function $y = f(x)$ from the given information only.

E. $(1, f(1))$ is an inflection point of the graph of the function $y = f(x)$.

15. Compute the limit

$$\lim_{x\to\left(\frac{\pi}{2}\right)^+} e^{\tan x}.$$

A. 0

B. ∞

C. e

D. $1/e$

E. 1

16. Compute the limit

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{9-\left(\frac{3i}{n}\right)^2}\cdot\frac{3}{n}.$$

A. 9π

B. $\frac{9\pi}{2}$

C. $\frac{9\pi}{4}$

D. $\frac{\pi}{2}$

E. π

17. Water is poured into a conical paper cup at a rate of 3 cm^3/s. Suppose that the cup is 12 cm tall and the top has a radius 6 cm. How fast is the water level rising when the water is 6 cm deep?

A. $\frac{1}{2\pi}$cm/s

B. $\frac{1}{3\pi}$cm/s

C. $\frac{2}{5\pi}$cm/s

D. $\frac{1}{\pi}$cm/s

E. $\frac{3}{2\pi}$cm/s

18. A box with square base and open top is required to have a volume of 8 m^3.

Find the base length which minimizes the surface area of the box.

A. $2^{1/3}$ m

B. $2^{4/3}$ m

C. $\frac{1}{2}$ m

D. $2^{-1/2}$ m

E. $\frac{1}{3}$ m

19. Compute the area between the graph of the function $y = x^2 \cos(x^3)$ and the x-axis over the interval $-(\frac{\pi}{2})^{1/3} \leq x \leq (\frac{\pi}{2})^{1/3}$.

A. $\frac{\pi}{2}$

B. 0

C. $\frac{2\pi}{3}$

D. $\frac{2}{3}$

E. π

20. The number of bacteria in a cell culture is initially observed to be 200. Two hours later the number is 300. Assuming that the bacteria grow exponentially, how many hours after the initial observation does the number of bacterial become equal to 500 ?

A. $\frac{\log 5+\log 2}{\log 2}$

B. $\frac{\log 5}{\log 3}$

C. $\frac{2(\log 5+\log 2)}{\log 3+\log 2}$

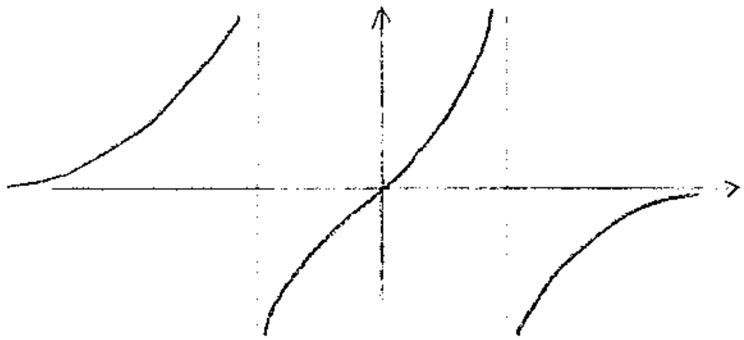
D. $\frac{2(\log 5-\log 2)}{\log 3-\log 2}$

E. $\frac{\log 2}{\log(\frac{3}{2})}$

21. Choose the one which describes best the graph of the function

$$f(x) = \frac{x}{x^2 - 9} \text{ over the interval } (-\infty, \infty).$$

A.



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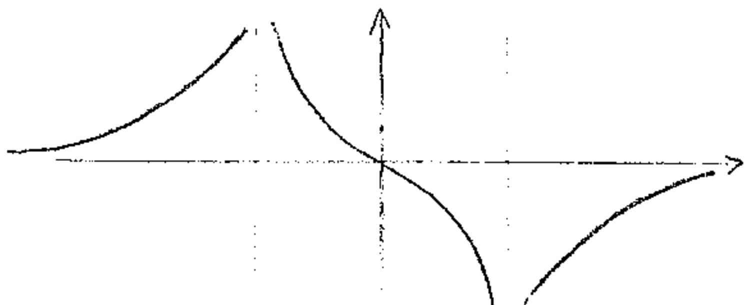

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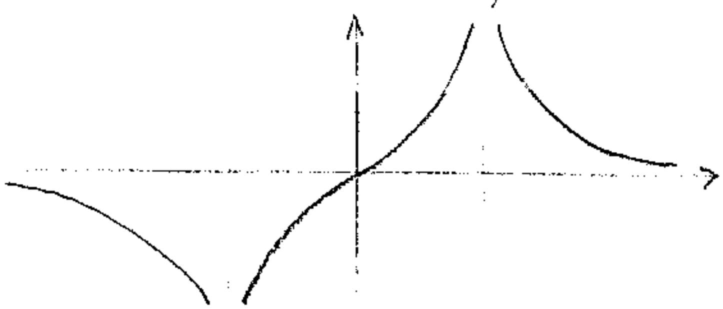

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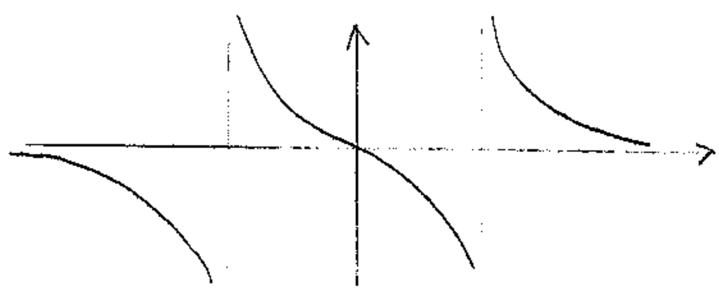
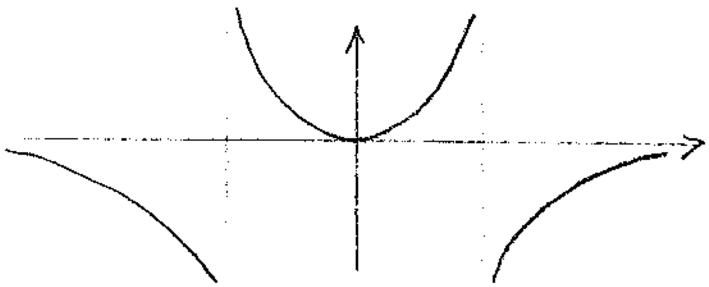


E.



22. Compute $\frac{d}{dx}\int_{x^2}^{x^3} e^{t^2}dt$

A. e^{x^6}

B. $e^{x^6} - e^{x^4}$

C. $3x^3e^{x^6}$

D. $3x^3e^{x^6} - 2x^2e^{x^4}$

E. $3x^2e^{x^6} - 2xe^{x^4}$

23. A parabola with y-axis as its axis and vertex at the origin passes through the point $(8, 8)$. Which of the following is the focus of the parabola ?

A. $(0, 0)$

B. $(4, 0)$

C. $(0, 4)$

D. $(2, 0)$

E. $(0, 2)$

24. An ellipse has vertices $(\pm 3+1, 2)$ and foci $(\pm 1+1, 2)$. Which of the following is the equation of the ellipse?

A. $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$

B. $\frac{(x-1)^2}{8} + \frac{(y-2)^2}{9} = 1$

C. $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{8} = 1$

D. $\frac{(x-1)^2}{9} + (y-2)^2 = 1$

E. $(x-1)^2 + \frac{(y-2)^2}{9} = 1$

25. Which of the following is an asymptote of the hyperbola

$$4x^2 - 9y^2 + 16x + 18y + 43 = 0 \ ?$$

A. $3y + 2x + 1 = 0$

B. $4y - 3x - 5 = 0$

C. $5y + 4x - 7 = 0$

D. $4y - 5x + 3 = 0$

E. $5x + 3y - 6 = 0$