MA 16500
FINAL EXAM INSTRUCTIONS
VERSION 01
DECEMBER 12, 2012

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–25. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 25 questions, each worth 8 points. The maximum possible score is

8×25 (for taking the exam) $= 200$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 5:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 5:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Questions

1. Find the domain of a function $f(x) = \ln\left(1 - \sqrt{2 - x}\right)$.

A. $(1, 2)$

B. $[1, 2)$

C. $(1, 2]$ (correct)

D. $[1, 2]$

E. $(-\infty, 2]$

2. Compute

$$\lim_{x \to 0^+} \ln\left(\frac{3x^2 + \sqrt{x}}{x^2 - x + 2\sqrt{x}}\right).$$

A. $\ln 3$

B. ∞

C. $-\infty$

D. $-\ln 2$ (correct)

E. The limit does not exist.

3. Given $f(x) = \sqrt{1 - 2x^3}$, find the formula for its inverse function $f^{-1}(x)$.

A. $f^{-1}(x) = (2 - 2x^2)^{\frac{1}{3}}$

B. $f^{-1}(x) = \left(\frac{1+x^2}{2}\right)^{\frac{1}{3}}$

C. $f^{-1}(x) = \left(\frac{1-x^2}{2}\right)^{\frac{1}{3}}$ (correct)

D. $f^{-1}(x) = \left(\frac{1-x^2}{2}\right)^{3}$

E. $f^{-1}(x) = \left(\frac{1+x^2}{2}\right)^{3}$

4. We have the following description of a function

$$\begin{cases} \frac{x^2+x-2}{x-1} & \text{if } x < 1 \\ a + bx & \text{if } 1 \le x < 2 \\ ax + b - 1 & \text{if } 2 \le x. \end{cases}$$

Determine the values of a and b so that the function f is continuous everywhere.

A. $a = 2, b = 1$ (correct)

B. $a = 1, b = 2$

C. $a = 1, b = -2$

D. $a = -1, b = 1$

E. $a = -1, b = 2$

5. Compute $g'(1)$ when

$$g(x) = \ln\left(\frac{\sqrt{3x+1}}{2x-1}\right).$$

A. $\frac{3}{8}$

B. $-\frac{11}{8}$

C. $-\frac{13}{8}$ (correct)

D. $\frac{1}{4}$

E. $-\frac{5}{4}$

6. Determine the value of $F'(1)$ for $F(x) = f(g(h(x)))$, given the following information

$$\begin{cases} h(1) &= 2, & h(2) &= 3 \\ g(2) &= 3, & g(3) &= 2 \\ h'(1) &= 3, & h'(2) &= 2 \\ g'(3) &= 2, & g'(2) &= 4 \\ f'(3) &= 5, & f'(4) &= 3 \end{cases}$$

A. 30

B. 60 (correct)

C. 18

D. 24

E. 48

7. Compute $f'(\frac{\pi}{4})$ when

$$f(x) = (1+3x)^{\sin 2x}.$$

A. 3 (correct)

B. $\frac{3}{1+\frac{3\pi}{4}}$

C. 2

D. $\left(1+\frac{3\pi}{4}\right)^{1-\frac{\pi}{4}}$

E. $\left(1+\frac{3\pi}{4}\right) \cdot \ln\left(1+\frac{3\pi}{4}\right)$

8. Find the slope of the tangent line at the point $(1, 2)$ to the curve

$$xy^3 - 3x^2y = 2.$$

A. $\frac{1}{3}$

B. $-\frac{1}{3}$

C. $\frac{4}{15}$

D. $\frac{2}{9}$

E. $\frac{4}{9}$ (correct)

9. Use the linear approximation of a function $f(x) = \sqrt[3]{x}$ at $a = 8$ to estimate the number $\sqrt[3]{8.03}$.

A. 2.01

B. 2.005

C. 2.0075

D. 2.0025 (correct)

E. 2.015

10. Find the absolute maximum "Max" and absolute minimum "Min" of the function

$$f(x) = x + \frac{4}{x} \text{ on the interval } [1, 5].$$

A. Max $= 5$, Min $= 4$

B. Max $= \frac{29}{5}$, Min $= 4$ (correct)

C. Max $= \frac{29}{5}$, Min $= 5$

D. Max $= \frac{29}{5}$, but Min does not exist.

E. Max $= 5$, Min $= \frac{24}{5}$

11. We have a function $y = f(x)$ whose first derivative is given by the following formula

$$f'(x) = x(x-1)^2(x-2)^3(x-3)^4.$$

Determine the set "S_{LocalMax}" of all the values of x which give the local maximum, and "S_{LocalMin}" of all the values of x which give the local minimum.

A. $S_{\text{LocalMax}} = \{2, 0\}$, $S_{\text{LocalMin}} = \{1, 3\}$

B. $S_{\text{LocalMax}} = \{1, 3\}$, $S_{\text{LocalMin}} = \{0, 2\}$

C. $S_{\text{LocalMax}} = \{2\}$, $S_{\text{LocalMin}} = \{0\}$

D. $S_{\text{LocalMax}} = \{0\}$, $S_{\text{LocalMin}} = \{2\}$ (correct)

E. $S_{\text{LocalMax}} = \{1\}$, $S_{\text{LocalMin}} = \{3\}$

12. Suppose $\cosh(x) = \frac{5}{3}$ and $x > 0$. Find the value for $\tanh(x)$.

A. $\frac{3}{5}$

B. $-\frac{3}{5}$

C. $\frac{4}{5}$ (correct)

D. $-\frac{4}{5}$

E. $\frac{3}{4}$

13. Choose the one which describes best the graph of a function

$$f(x) = \frac{\cos x}{2 + \sin x} \text{ on } [0, 2\pi].$$

A.

B.

C.

D. (correct)

E.

14. The graph of the second derivative $f''(x)$ is given as follows:

How many inflection points do we have on the graph of the original function $f(x)$?

A. 0

B. 1

C. 2 (correct)

D. 3

E. Can not determine only from the given information.

15. Water is withdrawn from a conical reservoir, 10 feet in diameter and 6 feet deep (vertex down) at the constant rate of 5 ft^3/min. How fast is the water level falling when the depth of the water in the reservoir is 2 feet ?

A. $\frac{15}{16\pi}$ ft/min

B. $\frac{5}{4\pi}$ ft/min

C. $\frac{5}{9\pi}$ ft/min

D. $\frac{9}{5\pi}$ ft/min (correct)

E. $\frac{2}{\pi}$ ft/min

16. A rectangle is inscribed as shown below in the ellipse given by an equation

$$\frac{x^2}{9} + \frac{y^2}{25} = 1.$$

Calculate the area of the largest such rectangle.

A. 15

B. 30 (correct)

C. $\frac{15\pi}{2}$

D. 15π

E. $15\sqrt{2}$

17. Compute

$$\lim_{x\to\infty} x \tan\left(\frac{5}{x}\right).$$

A. ∞

B. 0

C. 5 (correct)

D. $\frac{1}{5}$

E. The limit does not exist.

18. Compute

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(3 + i \cdot \frac{5}{n}\right)^2 \cdot \frac{5}{n}$$

A. ∞

B. $\frac{55}{2}$

C. 55

D. $\frac{\pi}{4}$

E. $\frac{485}{3}$ (correct)

19. Find $h'(x)$ when

$$h(x) = \int_{-2x}^{2x} \cos^2(t)dt.$$

A. 0

B. $\cos(4x)$

C. $\cos^2(2x)$

D. $2\cos(2x)\sin(2x)$

E. $4\cos^2(2x)$ (correct)

20. Compute

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} dx.$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{6}$

C. $\sin^{-1}(\sqrt{3})$

D. $\frac{\pi}{3}$ (correct)

E. 1

21. Compute

$$\int_1^2 x\sqrt{x-1}dx.$$

A. $\frac{16}{15}$ (correct)

B. $-\frac{8}{15}$

C. $\frac{2}{15}$

D. $\frac{\pi}{4}$

E. $\frac{\pi}{2}$

22. The half-life of radium-226 is 1590 years. A sample of radium-226 has a mass of 100mg. When we write the formula for the mass of the sample after t years to be

$$m(t) = 100 \cdot e^{kt},$$

find the constant k.

A. $\frac{1590}{\ln 2}$

B. $-\frac{1590}{2}$

C. $\frac{\ln 2}{1590}$

D. $-\frac{\ln 2}{1590}$ (correct)

E. $\ln 2 \cdot 1590$

23. Consider the ellipse given by the following equation

$$16x^2 - 64x + 9y^2 - 18y = 71.$$

Denote the two foci of the ellipse by F_1 and F_2. Find $PF_1 + PF_2$ for a point P on the ellipse.

A. 3

B. 4

C. 6

D. 8 (correct)

E. It depends on the position of P, and we can not determine only from the given information.

24. Consider the hyperbola given by the following equation

$$\frac{(y-1)^2}{5^2} - \frac{(x+3)^2}{12^2} = 1.$$

Find the coordinates of the two foci.

A. $(3, 14), (3, -12)$
B. $(-3, 14), (-3, -12)$ (correct)
C. $(-3, 6), (-3, -4)$
D. $(10, 1), (-16, 1)$
E. $(16, 1), (-10, 1)$

25. Find the equation of the parabola whose focus is $(10, 0)$ and whose directrix is given by the equation $x = -6$.

A. $y^2 = 40x$
B. $y^2 = 32x$
C. $y^2 = 32x - 64$ (correct)
D. $y^2 = 32x + 64$
E. $x^2 = 32y - 64$