MA 16500
EXAM 3 INSTRUCTIONS
VERSION 01
November 7, 2022

Your name ____________________ Your TA's name ____________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces. Put 00 at the front of PUID to make it a 10 digit number, and then fill it in.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, 10 of which are worth 8 points and 2 of which are worth 10 points. The maximum possible score is

10 questions $\times$ 8 points $+$ 2 questions $\times$ 10 points $=$ 100 points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

Questions

1. (8 points) Consider the function $f(x) = xe^{2x}$.

Let M be the absolute maximum value of $f(x)$ and m be the absolute minimum value of $f(x)$ on the closed interval $[-1, 0]$.

What is $M + m$?

A. $-\dfrac{1}{e^2}$

B. $-\dfrac{1}{2e}$

C. $-\dfrac{e+2}{e^2}$

D. $-\dfrac{e^2+4}{2e^2}$

E. $-\dfrac{1}{e}$

2. Suppose that $f(x)$ has derivative

$$f'(x) = (x+2)x^3(x-1)^2(x-4).$$

How many local minima and local maxima does the graph of $y = f(x)$ have?

A. 1 local minimum, no local maximum

B. 1 local minimum, 1 local maximum

C. 2 local minima, 1 local maximum

D. 1 local minimum, 2 local maxima

E. 2 local minima, 2 local maxima

3. (8 points) The second derivative of the function f is given by

$$f''(x) = (x+3)^6(x+1)^5x^4(x-1)^3(x-3)^2$$

How many inflection points does the graph of $y = f(x)$ have?

A. 0

B. 1

C. 2

D. 3

E. 4

4. (8 points) Compute the following limit

$$\lim_{x \to 0} \frac{\ln\left(\cos(3x)\right)}{x^2}$$

A. 0

B. ∞

C. $3/2$

D. $-3/2$

E. $-9/2$

5. (8 points) Compute the following limit

$$\lim_{\theta \to \frac{\pi}{2}^-} (2\tan\theta)^{\cos\theta}.$$

A. ∞

B. 1

C. 2

D. e

E. $2e$

6. (8 points) Use the linear approximation to $f(x) = \sqrt[4]{x}$ at $a = 256 = 4^4$ to estimate $\sqrt[4]{255}$.

A. $4 - \dfrac{1}{4}$

B. $4 - \dfrac{1}{16}$

C. $4 - \dfrac{1}{64}$

D. $4 - \dfrac{1}{128}$

E. $4 - \dfrac{1}{256}$

7. (8 points) Evaluate

$$\tan^{-1}\left(\frac{2}{5}\right) - \tan^{-1}\left(-\frac{5}{2}\right).$$

HINT: Consider the function

$$f(x) = \tan^{-1}(x) - \tan^{-1}\left(-\frac{1}{x}\right)$$

over the open interval $(0, \infty)$. What can you say about $f'(x)$?

A. 0

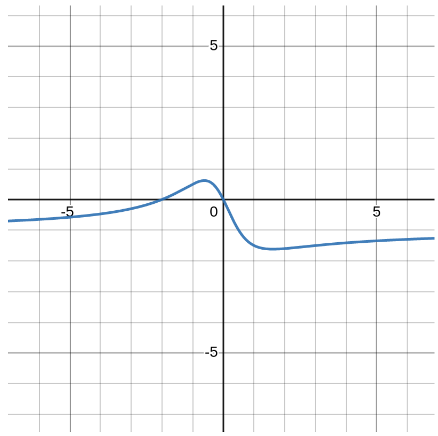
B. 1

C. $\pi/2$

D. $-\pi/2$

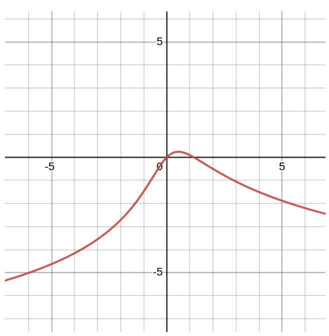
E. $\pi/4$

8. (8 points) Below is given the graph of $y = f'(x)$.



Which of the following can be the graph of $y = f(x)$.

A.



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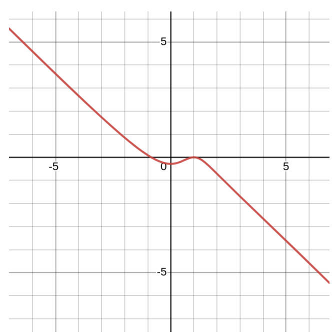
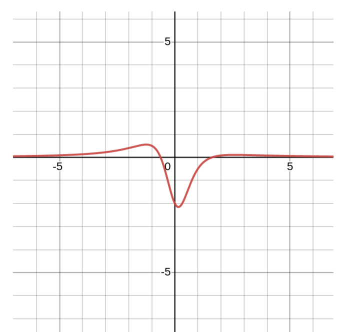


C.



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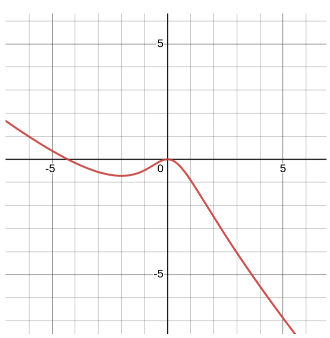
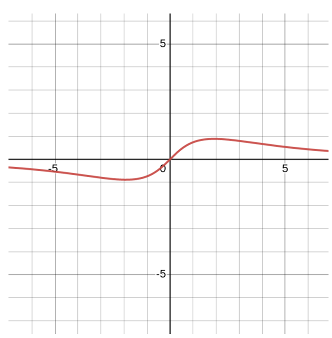


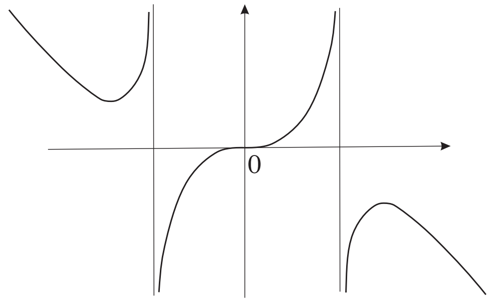
E.



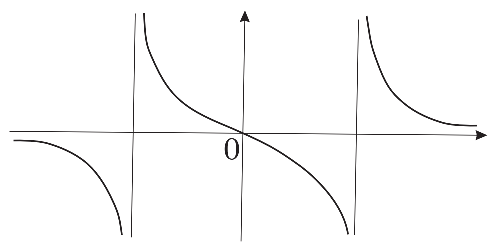
9. (8 points) Which of the following best describes the graph of the function

$$y = f(x) = \frac{x^3}{1-x^2} \ ?$$

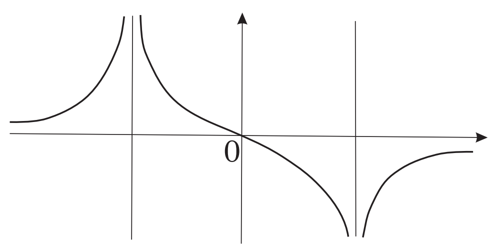
A.



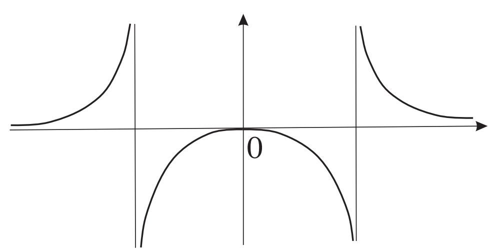
B.



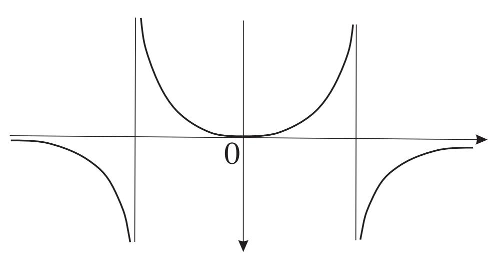
C.



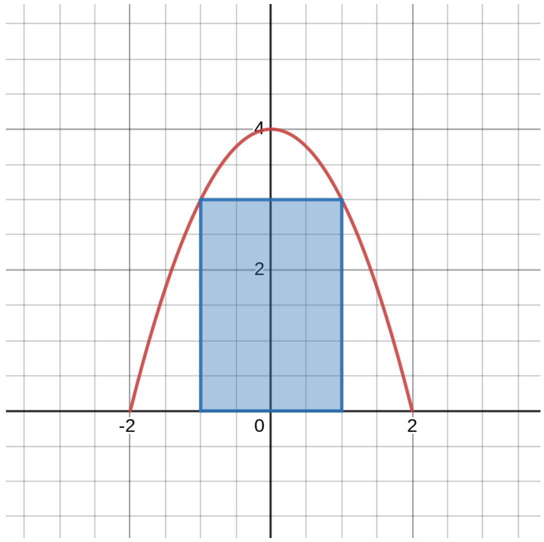
D.



E.



10. (8 points) What is the maximum area of a rectangle that lies between the curve $y = 4 - x^2$ and the x-axis as in the diagram below.



A. $\dfrac{32}{3\sqrt{3}}$

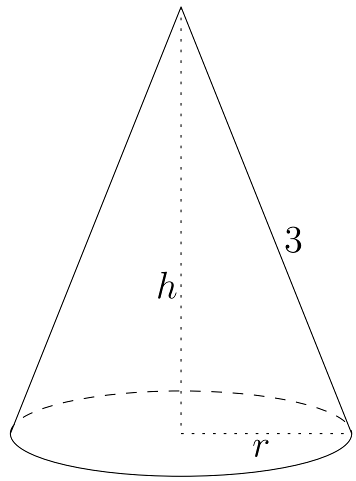
B. $4\sqrt{2}$

C. $9 - \sqrt{17}$

D. $\dfrac{13\sqrt{3}}{4}$

E. 6

11. (10 points) The slant height of a right circular cone is the distance from the edge of the base of the cone to the vertex of the cone.



What is the maximum volume of a right circular cone with slant height 3 cm.

A. $\dfrac{9\sqrt{3}\,\pi}{8}$ cm^3

B. $\dfrac{18\,\pi}{5\sqrt{5}}$ cm^3

C. $\dfrac{9\,\pi}{2\sqrt{2}}$ cm^3

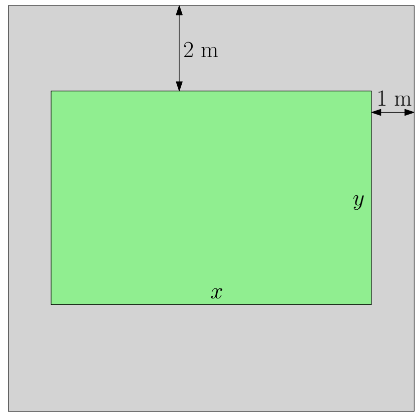
D. $2\sqrt{3}\,\pi$ cm^3

E. $3\sqrt{3}\,\pi$ cm^3

HINT: The volume V of a right circular cone with radius r and height h as illustrated in the picture above is given by

$$V = \frac{1}{3}\pi r^2 h.$$

12. (10 points) Suppose a rectangular area is to be surrounded by a concrete walkway that is 1 meter wide on the East and West and 2 meters wide on the North and South.



If the area inside the walkway is to be 100 square meters, what should the interior width of the enclosed area be (labeled x in the diagram above) in order to minimize the amount of concrete used.
(Note minimizing the amount of concrete used is equivalent to minimizing the area covered by concrete.)

A. $x = 2\sqrt{5}$

B. $x = 5\sqrt{2}$

C. $x = 10\sqrt{2}$

D. $x = 7$

E. $x = 10$