MA 16500
EXAM 3 INSTRUCTIONS
VERSION 01
November 11, 2019

Your name ________________ Your TA's name ________________

Student ID # ________________ Section # and recitation time ________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While **mark all your work on the scantron sheet**, you should **show your work on the exam booklet**. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. Find the absolute minimum "min" and absolute maximum "Max" of the function

$$f(x) = 2\cos(x) + \sin(2x) \text{ over } [0, \pi/2].$$

A. $\min = 0, \mathrm{Max} = 2$

B. $\min = 0, \mathrm{Max} = \dfrac{\sqrt{3}}{2}$

C. $\min = 0, \mathrm{Max} = \dfrac{3\sqrt{3}}{2}$

D. $\min = \dfrac{\sqrt{3}}{2}, \mathrm{Max} = 2$

E. $\min = 2, \mathrm{Max} = \dfrac{3\sqrt{3}}{2}$

2. The function $f(x) = (x^2 - 1)^3$ has:

A. one local min., no local max., two inflection points.

B. one local min., two local max., two inflection points.

C. one local min., no local max., four inflection points.

D. one local min., two local max., four inflection points.

E. two local min., one local max., two inflection points.

3. Find the following limits:

(a) $\lim_{x\to 0} \dfrac{e^{2x} - \cos x}{\tan(5x)}$

(b) $\lim_{x\to \pi^-} \dfrac{\sin x}{1 - \cos x}$

A. (a) 0 (b) 0

B. (a) DNE (b) 0

C. (a) $\dfrac{2}{5}$ (b) $+\infty$

D. (a) $\dfrac{2}{5}$ (b) 0

E. (a) $\dfrac{2}{5}$ (b) $-\infty$

4. Find the following limits:

(a) $\lim_{x\to\infty} \left(\dfrac{x+2}{x-1}\right)^x$

(b) $\lim_{x\to\infty} x^2 \sin\left(\dfrac{1}{3x^2+1}\right)$.

A. (a) e^3 (b) $\dfrac{1}{3}$

B. (a) e^3 (b) 0

C. (a) DNE (b) ∞

D. (a) e^{-3} (b) $\dfrac{1}{3}$

E. (a) e (b) 0

5. Use the linear approximation to estimate $\sqrt[3]{1001}$.

A. $10 + \dfrac{1}{30}$

B. $10 + \dfrac{1}{90}$

C. $10 + \dfrac{1}{150}$

D. $10 + \dfrac{1}{300}$

E. $10 + \dfrac{1}{600}$

6. We have a function $f(x)$ whose first derivative is given by the formula

$$f'(x) = x^2(x+2)^3.$$

Choose the correct statement from below.

A. The function has a local minimum at $x = -2$. Even though it also has a local extreme value at $x = 0$, we cannot determine if it is a local minimum or maximum from the given information. It has inflection points at $x = -2, 0$ and nowhere else.

B. The function has a local minimum at $x = -2$, and this is the only place where it has a local extreme value. It has inflection points at $x = -\frac{4}{5}, 0$ and nowhere else.

C. The function has local extrema at $x = -2, 0$ and nowhere else. It has inflection points at $x = -2, -\frac{4}{5}, 0$ and nowhere else.

D. We need to know the formula for the original function to determine where the function has local extrema and/or inflection points. Therefore, the given information is insufficient to conclude anything about the local extrema or inflection points.

E. The function has a local minimum at $x = -2$, and this is the only place where it has a local extreme value. It has inflection points at $x = -2, -\frac{4}{5}, 0$ and nowhere else.

7. Consider the function

$$f(x) = \frac{2x^3}{x^2+1}.$$

Determine

(a) the number of the local extrema,

(b) the number of the inflection points,

(c) the equation of the (slant) asymptote(s).

A. (a) 3 (b) 4 (c) $y = 0$ (i.e., x-axis)

B. (a) 1 (b) 3 (c) $y = \pm 2x$

C. (a) 3 (b) 5 (c) $y = 2x$

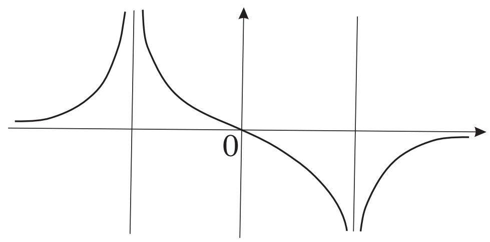
D. (a) 0 (b) 3 (c) $y = 2x$

E. (a) 1 (b) 4 (c) $y = 2x$

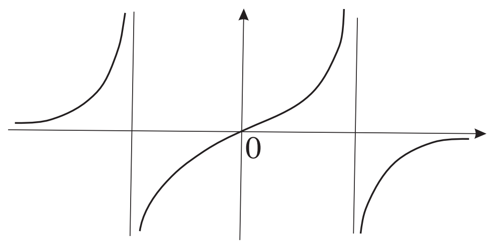
8. Which of the following best describes the graph of the function

$$y = f(x) = \frac{-x}{x^2 - 1} \ ?$$

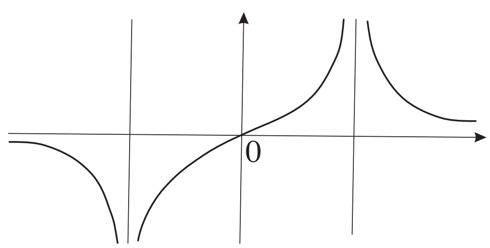
A.



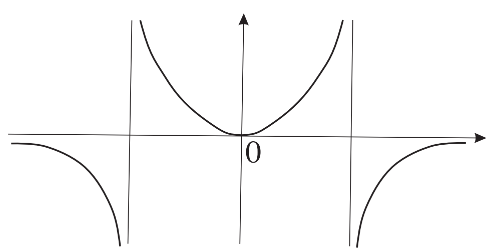
B.



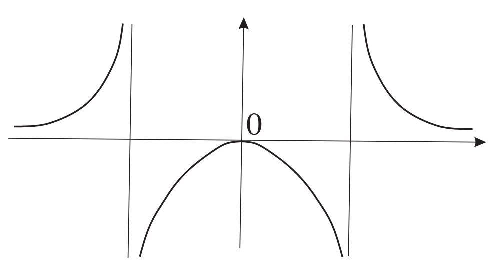
C.



D.



E.



9. Consider the function

$$f(x) = \sqrt[3]{x}$$

over the interval $[-1, 1]$.

Choose the correct statement from below.

A. Since f is continuous on $[-1, 1]$, differentiable over $(-1, 1)$, and since $\dfrac{f(1) - f(-1)}{1 - (-1)} = 1$, the Mean Value Theorem says there exists $c \in (-1, 1)$ such that $f'(c) = 1$. In fact, we can take $c = \pm\dfrac{\sqrt{3}}{9}$.

B. Since f is continuous on $[-1, 1]$, differentiable over $(-1, 1)$, and since $\dfrac{f(1) - f(-1)}{1 - (-1)} = 1$, the Mean Value Theorem says at the mean (average) point of -1 and 1, i.e., $c = \dfrac{(-1) + 1}{2} = 0$, we find $f'(c) = 1$.

C. Even though f is continuous on $[-1, 1]$, f is not differentiable over $(-1, 1)$. We compute $\dfrac{f(1) - f(-1)}{1 - (-1)} = 1$. Therefore, as the negation of the Mean Value Theorem we conclude that there does not exist any $c \in (-1, 1)$ such that $f'(c) = 1$.

D. Even though f is continuous on $[-1, 1]$, f is not differentiable over $(-1, 1)$. So we cannot apply the Mean Value Theorem to this situation.

E. Even though f is continuous on $[-1, 1]$, f is not differentiable over $(-1, 1)$. We compute $\dfrac{f(1) - f(-1)}{1 - (-1)} = 1$. But we still observe that $f'(c) = 1$ for $c = \pm\dfrac{\sqrt{3}}{9} \in (-1, 1)$. So we conclude that the Mean Value Theorem is false in this case.

10. A metal can has the shape of a circular cylinder with a bottom and a top. If the volume of the can V is fixed, which ratio of the height h to the radius r of the can requires the least amount of metal ?

A. 2

B. $\sqrt[3]{2}$

C. $1/2$

D. $1/\sqrt[3]{2}$

E. The ratio depends on the value of the volume V. Without knowing the value of V, we cannnot determine the ratio.

11. What is the largest area of an isosceles triangle inscribed in a circle of radius 2 ?

A. 3

B. $3\sqrt{3}$

C. 8

D. $2\sqrt{2}$

E. 2π

12. Find an equation of the line through the point $(7, 4)$ that cuts off the least area from the first quadrant.

A. $y = -\frac{\sqrt{3}}{3}x + \frac{7\sqrt{3}+12}{3}$

B. $y = -\frac{1}{2}x + \frac{15}{2}$

C. $y = -x + 11$

D. $y = -\frac{4}{7}x + 8$

E. $y = -\frac{7}{4}x + \frac{65}{4}$