MA 16500
EXAM 2 INSTRUCTIONS
VERSION 01
October 19, 2023

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, 10 of which are worth 8 points and 2 of which are worth 10 points. The maximum possible score is

$$10 \text{ questions } \times 8 \text{ points } + 2 \text{ questions } \times 10 \text{ points } = 100 \text{ points.}$$

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

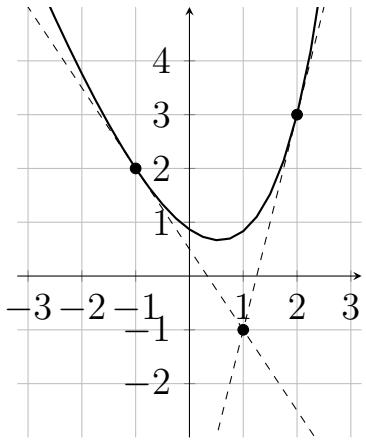
I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

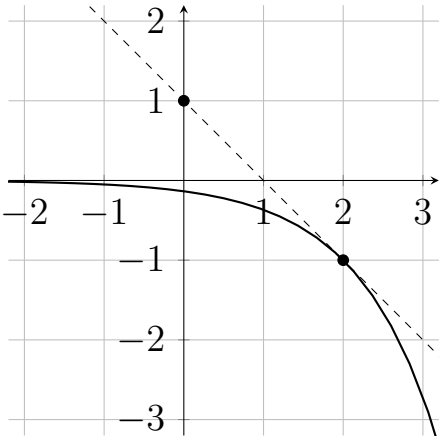
STUDENT SIGNATURE: _____

Questions

1. (8 points) Let $f(x)$ and $g(x)$ be two differentiable functions with the following graphs.



Graph of $f(x)$



Graph of $g(x)$

The dashed lines in the figures are tangent to the graphs at the indicated points. What is the derivative of $f(g(x))$ at $x = 2$?

A. -1

B. 3

C. -3

D. $\frac{3}{2}$ (CORRECT)

E. $-\frac{3}{2}$

2. (8 points) Let $h(x) = f\left(g\left(\frac{x^2}{2}\right)\right)$.

Assume that $g'(2) = \frac{1}{4}$, $g(2) = 3$, and $f'(3) = 9$.

Compute $h'(2)$.

A. 9

B. $\frac{9}{2}$ (CORRECT)

C. $\frac{9}{4}$

D. $\frac{27}{4}$

E. 27

3. A particle moves along a coordinate axis in such a way that its position is described by

$$s(t) = 2\cos(t) + \sqrt{3}t$$

for $0 < t < 2\pi$. At what time(s) t in between 0 and 2π is the particle's acceleration equal to $\sqrt{3}$?

Select the answer that includes *all* such time(s).

A. $t = \pi$

B. $t = \dfrac{\pi}{3}$ and $t = \dfrac{2\pi}{3}$

C. $t = \dfrac{\pi}{6}$ and $t = \dfrac{11\pi}{6}$

D. $t = \dfrac{5\pi}{6}$ and $t = \dfrac{7\pi}{6}$ (CORRECT)

E. $t = \dfrac{\pi}{2}$ and $t = \dfrac{3\pi}{2}$

4. (8 points) What is the slope of the line tangent to the curve implicitly defined by

$$x^3 + y^3 + 4xy = 7x + 6$$

in the xy-plane at the point $(-1, 2)$?

A. -2

B. -4

C. $-\frac{1}{2}$ (CORRECT)

D. 0

E. The tangent line is vertical.

5. (8 points) Compute the following limit

$$\lim_{h\to 0} \frac{(2+4h)^{2-2h}-4}{h}.$$

A. $16-8\ln 2$ (CORRECT)

B. $12+18\ln 2$

C. $4-\ln 2$

D. $8+\dfrac{1}{2}\ln 2$

E. $32-4\ln 2$

6. (8 points) Which of the following is the result of differentiating

$$f(x) = \ln(x)^{\ln(x)} \ ?$$

A. $f'(x) = x^{-1/x}$

B. $f'(x) = \dfrac{\ln(x)^{\ln(x)}}{x}$

C. $f'(x) = \dfrac{\ln(\ln(x))\ln(x)^{\ln(x)}}{x}$

D. $f'(x) = \dfrac{1+\ln(\ln(x))}{x}\ln(x)^{\ln(x)}$ (CORRECT)

E. $f'(x) = \dfrac{\ln(x)+\ln(\ln(x))}{x}\ln(x)^{\ln(x)}$

7. (8 points) The function $f(x) = 10x + \sin\left(\frac{\pi}{2}x\right)$ is one-to-one and hence it has an inverse function $f^{-1}(x)$. Observe that the point $(1, 11)$ is on the graph of $y = f(x)$.

Determine the equation of the tanget line to the graph of the inverse function

$$y = f^{-1}(x) \quad \text{at the point (11,1).}$$

A. $y = \frac{1}{11}x - \frac{\pi}{2}$

B. $y = \frac{2x - 1}{5\pi}$

C. $y = \frac{2x - 2 + 11\pi}{2 - \pi}$

D. $y = \frac{x - 109}{10}$

E. $y = \frac{x - 1}{10}$ (CORRECT)

8. (8 points) Which of the following is the result of differentiating

$$f(x) = \sin(\cos^{-1}(x)) \ ?$$

A. $f'(x) = x$

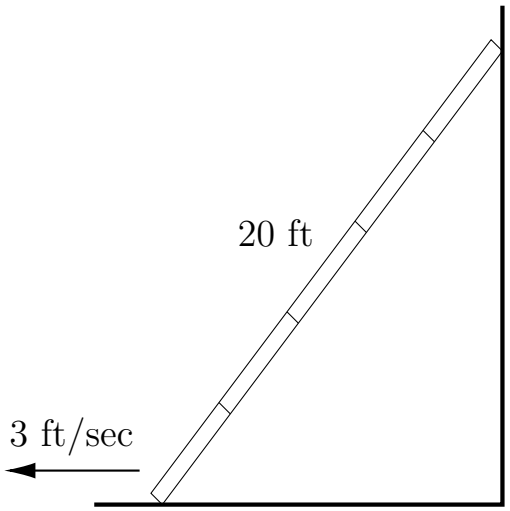
B. $f'(x) = -x$

C. $f'(x) = \sqrt{1 - x^2}$

D. $f'(x) = \dfrac{1}{2\sqrt{1 - x^2}}$

E. $f'(x) = -\dfrac{x}{\sqrt{1 - x^2}}$ (CORRECT)

9. (9 points) A 20 ft long ladder is leaned against a wall. The base of the ladder slides away from the wall at a rate of 3 ft/sec, how fast is the top of the ladder sliding down the wall when the base of the ladder is 16 ft from the wall?



A. 2 ft/sec

B. 4 ft/sec (CORRECT)

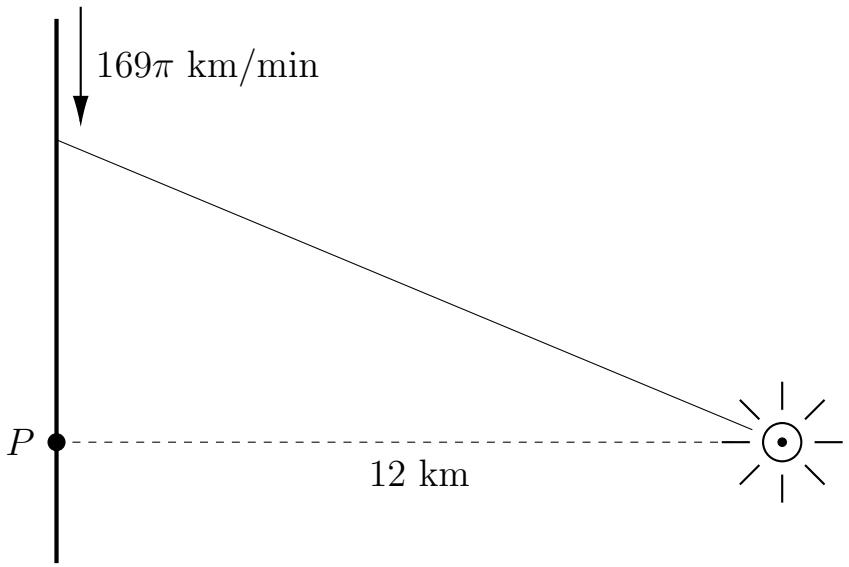
C. $\frac{4}{3}$ ft/sec

D. $\frac{9}{2}$ ft/sec

E. $\frac{3}{2}$ ft/sec

10. (9 points) A lighthouse is located on an island 12 km away from the nearest point P on a straight shoreline. The beam of light is moving along the shoreline at the speed of 169 π km/min when it is 5 km away from the point P.

How many revolutions per minute is the beam making at the lighthouse ?



A. 3

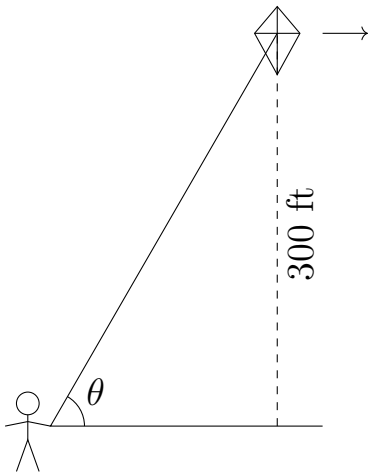
B. 4

C. 5

D. 6 (CORRECT)

E. 7

11. (9 points) A kite 300 ft above the ground flies horizontally away from the person holding the kite's string. Assume that the string is stretched into a straight line. At a particular moment in time, the elevation angle θ of the kite (the angle between the string and the ground) is $\pi/3$, and the elevation angle is decreasing at a rate of 0.01 rad/s. At this moment, what is the speed of the kite?



A. 1 ft/s

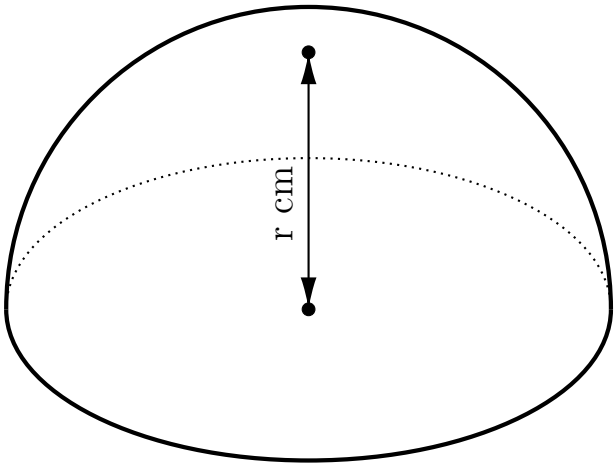
B. 2 ft/s

C. 3 ft/s

D. 4 ft/s (CORRECT)

E. 0.25 ft/s

12. (9 points) A lump of bread dough shaped like a hemisphere is baking in the oven. As the bread bakes, the hemisphere grows in size. At 10 minutes after baking starts, the bread is 3 cm high, and the height of the bread is rising at 0.5 cm/min. How fast is the volume of the bread changing at this moment?



NOTE: The hemisphere is HALF of a sphere.

Accordingly, the volume of a hemisphere is HALF of the volume of a sphere. Hence the volume of a hemisphere of radius r (which is also the height) is given by

$$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3.$$

A. 9π cm^3/min (CORRECT)

B. 27π cm^3/min

C. $\dfrac{27\pi}{2}$ cm^3/min

D. $\dfrac{27\pi}{4}$ cm^3/min

E. 90π cm^3/min