MA 16500
EXAM 2 INSTRUCTIONS
VERSION 01
October 19, 2022

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ___________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces. Put 00 at the front of PUID to make it a 10 digit number, and then fill it in.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, 10 of which are worth 8 points and 2 of which are worth 10 points. The maximum possible score is

10 questions $\times$ 8 points $+$ 2 questions $\times$ 10 points $=$ 100 points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

Questions

1. Find the derivative $\dfrac{dy}{dx}$ when $y = e^{2^{x^2}}$.

A. $\dfrac{dy}{dx} = 2^{x^2} e^{2^{x^2}}$

B. $\dfrac{dy}{dx} = 2^{x^2-1} e^{2^{x^2}}$

C. $\dfrac{dy}{dx} = x\, 2^{x^2-1} e^{2^{x^2}}$

D. $\dfrac{dy}{dx} = (\ln 2)\, x\, 2^{x^2} e^{2^{x^2}}$

E. $\dfrac{dy}{dx} = (\ln 4)\, x\, 2^{x^2} e^{2^{x^2}}$

CLARIFICATION: The power on the right hand shoulder of e in $y = e^{2^{x^2}}$ above is 2^{x^2}, which is "2 to the power of x^2", and NOT "$2x^2$", which is "2 times x^2.

2. (8 points) Set $y = \ln\left(\dfrac{e^x + 1}{e^x - 1}\right)$.

Compute $\dfrac{dy}{dx}$.

A. $\ln\left(\dfrac{2e^x}{(e^x - 1)^2}\right)$

B. $\dfrac{-2e^x}{(e^x - 1)^2}$

C. $\dfrac{e^x - 1}{e^x + 1}$

D. $\dfrac{2}{e^{2x} - 1}$

E. $\dfrac{-2e^x}{e^{2x} - 1}$

3. Suppose that $F(x) = f(x)f(g^{-1}(x))$ and that the functions f and g satisfy the following conditions:

$$\begin{cases} f(1) = 5, & f(2) = 3, & f(3) = -1 \\ f'(1) = 4, & f'(2) = 3, & f'(3) = -2 \\ g(1) = 3, & g(2) = 2, & g(3) = 1 \\ g'(1) = 2, & g'(2) = 3, & g'(3) = -2 \end{cases}$$

Find $F'(1)$.

A. 1

B. -1

C. 2

D. -2

E. -9

4. (8 points) What is the slope of the tangent line to the curve

$$6 + 2\sqrt{xy+1} = x^2 + y^2$$

at the point $(3, 1)$?

A. -13

B. -2

C. -11

D. 5

E. $\sqrt{2}$

5. (8 points) What is the slope of the tangent line to the curve

$$y = x^{\cos(x)},$$

when $x = \pi$.

A. $\dfrac{1}{\pi}$

B. $\dfrac{-1}{\pi^2}$

C. $\dfrac{\ln(\pi)}{\pi}$

D. $\dfrac{\pi \ln(\pi) - 1}{\pi^2}$

E. $\dfrac{-1}{\pi}$

6. (8 points) Compute the following limit

$$\lim_{h \to 0} \frac{(3+h)^{2-h} - 9}{h}.$$

A. $9 \ln 3$

B. $18 \ln 2$

C. $-18 \ln 3 + 2$

D. $9 \ln 2 + 6$

E. $-9 \ln 3 + 6$

7. (8 points) Compute $\dfrac{dy}{dx}$ when $y = \sin^{-1}(\sqrt{x})$.

A. $\dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{2\sqrt{x}}$

B. $\dfrac{1}{2\sqrt{x-x^2}}$

C. $\dfrac{1}{2\sqrt{x-x^3}}$

D. $\dfrac{\cos^{-1}(\sqrt{x})}{2\sqrt{x}}$

E. $\dfrac{1}{2}\sqrt{\dfrac{1-x^2}{x}}$

8. (8 points) The function $f(x) = 4x + \cos(\pi x)$ is one-to-one and hence it has an inverse function $f^{-1}(x)$. Observe that the point $(1, 3)$ is on the graph of $y = f(x)$.

Determine the equation of the tanget line to the graph of the inverse function

$$y = f^{-1}(x) \quad \text{at the point (3,1).}$$

A. $y = 4x - 11$

B. $y = \dfrac{2x - 1}{5}$

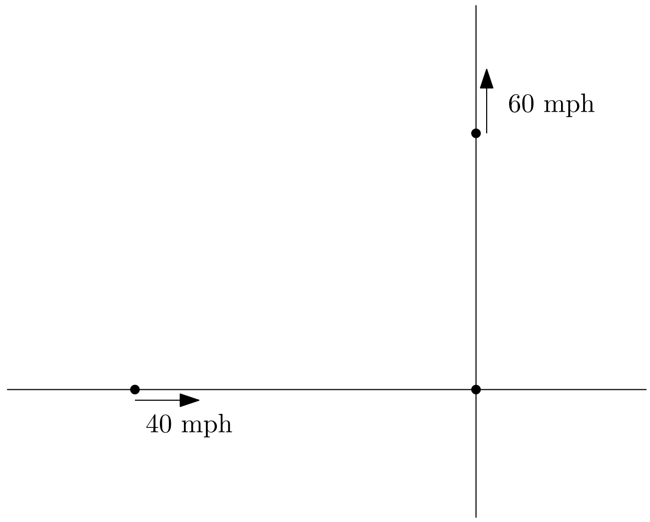
C. $y = \dfrac{2x - 2 - \pi}{4 - \pi}$

D. $y = \dfrac{x + 1}{4}$

E. $y = \dfrac{-x + 7}{4}$

9. (9 points) Aaron is driving north at 60 mph (miles per hour), and Becky is driving east on a different road at 40 mph.

If Aaron is currently 6 miles north of the intersection where the roads meet and Becky is 8 miles west of the intersection, how fast is the distance between them changing?



A. increasing at 4 mph

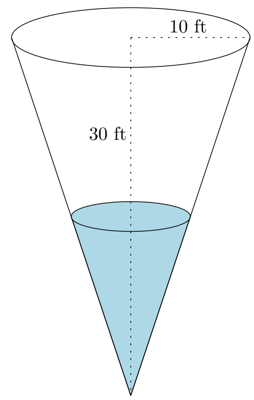
B. decreasing at 4 mph

C. increasing at 2 mph

D. decreasing at 2 mph

E. increasing at 6 mph

10. (9 points) Water is being drained out of a conical tank with a radius of 10 ft and a height of 30 ft.



If the water is being drained at a constant rate of 100 ft^3/min, how fast is the depth of the water in the tank decreasing at the instant when the depth is 5 ft deep.

HINT: The volume V of a reversed circular cone with radius r for the top circle and height h is given by

$$V = \frac{1}{3}\pi r^2 h.$$

A. $\dfrac{36}{\pi}$ ft/min

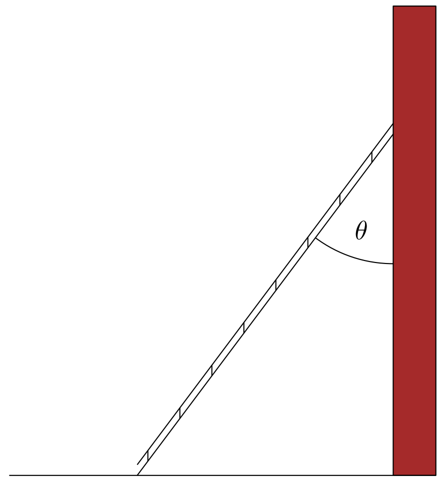
B. $\dfrac{2500\pi}{9}$ ft/min

C. $\dfrac{30}{\pi^2}$ ft/min

D. $\dfrac{12}{\pi}$ ft/min

E. $\dfrac{18}{\pi}$ ft/min

11. (9 points) A 5 ft long ladder is leaned against a wall. The base of the ladder is being pulled away from the wall at a rate of 2 ft/sec.



If θ is the angle formed by the ladder with the wall, how fast is θ changing (measured in rad/sec, i.e., radians per second) when the base of the ladder is 3 ft from the wall.

A. 2 rad/sec

B. 4 rad/sec

C. $\frac{2}{5}$ rad/sec

D. $\frac{1}{2}$ rad/sec

E. $\frac{5}{2}$ rad/sec

12. (9 points) Suppose the volume of a sphere is increasing at a constant rate of 10 in^3/sec. What will the radius of the sphere be at the instant when the radius of the sphere is increasing at 5 in/sec?

HINT: The volume V of a sphere of radius r is given by:

$$V = \frac{4}{3}\pi r^3.$$

A. $\dfrac{1}{2\pi}$ in

B. $\dfrac{1}{\sqrt{2\pi}}$ in

C. $\dfrac{2}{\sqrt{3\pi}}$ in

D. $\dfrac{2}{\pi^{1/3}}$ in

E. $\dfrac{1}{(2\pi)^{1/3}}$ in