MA 16500
EXAM 2 INSTRUCTIONS
VERSION 01
October 15, 2018

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your work on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, each worth 8 points. The maximum possible score is
$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 8:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 8:55, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Questions

1. Find the exact values for

(a) $\tan^{-1}(\tan(2))$

(b) $\sin\left(2\tan^{-1}\left(\frac{3}{4}\right)\right)$

Warning: The range of the function $\tan^{-1}$ is $(-\pi/2, \pi/2)$.

A. (a) 2 (b) $-\frac{3}{16}$

B. (a) $2-\pi$ (b) $\frac{4}{25}$

C. (a) $\pi-2$ (b) $\frac{8}{25}$

D. (a) $2-\pi$ (b) $\frac{24}{25}$

E. (a) $\pi-2$ (b) $-\frac{24}{25}$

2. Compute the derivative of

$$f(x) = x^{1/x}.$$

A. $f'(x) = x^{\frac{1}{x}-2}$

B. $f'(x) = x^{\frac{1}{x}-1}$

C. $f'(x) = x^{\frac{1}{x}-2}\,(1 - \ln(x))$

D. $f'(x) = x^{\frac{1}{x}}\,(1 + \ln(x))$

E. $f'(x) = -x^{\frac{1}{x}}(1 + \ln(x))$

3. Compute the slope of the tangent line for the curve

$$(x^2 + 2y^2)^2 = x^2 - 2y$$

at the point $(1, 0)$.

A. -3

B. -1

C. 0

D. 2

E. 3

4. Use linear approximation of the function $f(x) = x^{10}$ at $x = 2$ to approximate $(2.01)^{10}$.

HINT: It may be helpful to note that $2^{10} = 1024$.

A. 1025

B. 1030.12

C. 1034.24

D. 1075.2

E. 1537

5. Compute the derivative of

$$f(x) = \sin^{-1}(\sqrt{1-x}).$$

A. $f'(x) = \dfrac{-1}{\sqrt{x}}$

B. $f'(x) = \dfrac{1}{2\sqrt{(1-x^2)(1-x)}}$

C. $f'(x) = \dfrac{-1}{2\sqrt{(1-x^2)(1-x)}}$

D. $f'(x) = \dfrac{1}{2\sqrt{x(1-x)}}$

E. $f'(x) = \dfrac{-1}{2\sqrt{x(1-x)}}$

6. Suppose that the position of a particle along a straight line is given by

$$s = \frac{1}{3}t^3 - 3t^2 + 5t - 5$$

for $0 \leq t \leq 10$.

Find all the interval(s) of t during which the particle is slowing down.

A. $(1, 3)$, $(5, 10)$

B. $(0, 3)$

C. $(0, 1)$, $(3, 5)$

D. $(1, 5)$

E. $(0, 1)$, $(5, 10)$

7. Let $f(x) = \cosh(\ln x)$. Evaluate $f(5)$ and $f'(5)$.

A. $f(5) = \dfrac{13}{5}$, $f'(5) = \dfrac{2 + \ln 5}{\ln 5}$

B. $f(5) = \dfrac{13}{5}$, $f'(5) = \dfrac{12}{25}$

C. $f(5) = \dfrac{3}{\ln 5}$, $f'(5) = \dfrac{12 \ln 5}{25}$

D. $f(5) = \dfrac{3}{\ln 5}$, $f'(5) = \dfrac{6 \ln 5}{25}$

E. $f(5) = \dfrac{\ln 5}{5}$, $f'(5) = \dfrac{3}{25}$

8. Find the derivative of the following function

$$y = \ln(e^{-x} + xe^{-x}).$$

A. $\dfrac{dy}{dx} = \dfrac{x}{1+x}$

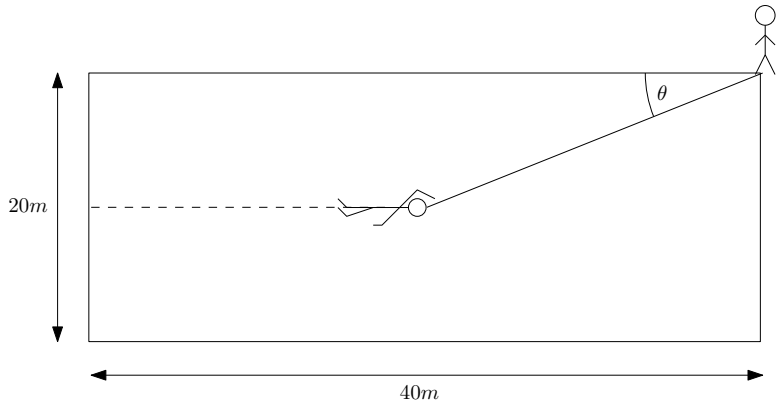
B. $\dfrac{dy}{dx} = -\dfrac{x}{1+x}$

C. $\dfrac{dy}{dx} = \dfrac{1}{1+x}$

D. $\dfrac{dy}{dx} = \dfrac{e^{-x}(2+x)}{e^{-x}+xe^{-x}}$

E. $\dfrac{dy}{dx} = -x + \ln(1+x)$

9. Fred is swimming a race in a pool 40 m long and 20 m wide. He is swimming his race in the center lane so that he is always 10 m from either side. His coach is standing at the corner of the pool by the finish line. Suppose Fred swims his race at a constant rate of 2 m/s. Let θ be the angle between the side of the pool and the line between Fred and his coach. What is the rate at which θ is changing (in radians per second) when Fred is halfway through the 40m race?



A. $\dfrac{d\theta}{dt} = \dfrac{1}{20}$ rad/sec

B. $\dfrac{d\theta}{dt} = \dfrac{1}{10\sqrt{5}}$ rad/sec

C. $\dfrac{d\theta}{dt} = \dfrac{2}{\sqrt{5}}$ rad/sec

D. $\dfrac{d\theta}{dt} = \dfrac{1}{25}$ rad/sec

E. $\dfrac{d\theta}{dt} = \dfrac{\pi}{6}$ rad/sec

10. Suppose that a water tank has the shape of an inverted circular cone with radius 1 m and height 5 m. When the water is 2 m deep in the tank, the depth of the water is increasing at a rate of 0.5 m/sec.

What is the rate at which the volume of the water in the tank is increasing at the same time ?

A. $\frac{\pi}{600}$ $\mathrm{m}^3/\mathrm{sec}$

B. $\frac{\pi}{300}$ $\mathrm{m}^3/\mathrm{sec}$

C. $\frac{2\pi}{25}$ $\mathrm{m}^3/\mathrm{sec}$

D. $\frac{4\pi}{75}$ $\mathrm{m}^3/\mathrm{sec}$

E. $\frac{8\pi}{75}$ $\mathrm{m}^3/\mathrm{sec}$

11. The lamp of a street light is 36 ft above ground. A man 6 ft tall walks away from the street light. If the length of the man's shadow is increasing at the rate of 1 ft/sec, how fast is he walking?

A. $\frac{5}{2}$ft/sec

B. 5ft/sec

C. 6ft/sec

D. 2ft/sec

E. 3ft/sec

Warning: Make a clear distinction between

- the rate at which the length of the shadow is increasing, and
- the rate at which the tip of the shadow is moving away from the street light.

12. At noon, ship A is 20 km west of ship B. Ship A is sailing east at 2 km/hr and ship B is sailing north at 3 km/hr. What is the rate of change of the distance between the ships at 4:00 PM ?

A. 2 km/hr

B. $\sqrt{2}$ km/hr

C. $\frac{\sqrt{2}}{2}$ km/hr

D. $\frac{5\sqrt{2}}{2}$ km/hr

E. $12\sqrt{2}$ km/hr