

MA 16500
EXAM 1 INSTRUCTIONS
VERSION 01
September 16, 2019

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).
2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.
3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.
4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.
5. Fill in the four-digit SECTION NUMBER.
6. Sign the scantron sheet.
7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While mark all your work on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.
8. There are 12 questions, each worth 8 points. The maximum possible score is $8 \times 12 + 4$ (for taking the exam) = 100 points.
9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.
10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.
11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Questions

1. Consider the function

$$f(x) = \left(e^{|x-5|}\right)^2 + 3$$

over the domain $[0, 1]$.

Choose the correct statement about its inverse function from below.

- A. f is not one-to-one over the specified domain, and hence the inverse function f^{-1} does not exist.
- B. The formula for the inverse function is $f^{-1}(x) = -\frac{\ln(x-3)}{2} + 5$, and its domain is $[e^8 + 3, e^{10} + 3]$.
- C. The formula for the inverse function is $f^{-1}(x) = \frac{\ln(x-3)}{2} + 5$, and its domain is $[e^8 + 3, e^{10} + 3]$.
- D. The formula for the inverse function is $f^{-1}(x) = \frac{\ln(x-3)}{2} - 5$, and its domain is $[e^{10} + 3, e^8 + 3]$.
- E. The formula for the inverse function is $f^{-1}(x) = |\ln \sqrt{x-3}| + 5$, and its domain is $[e^8 + 3, e^{10} + 3]$.

2. Choose the right answers for the following questions (a) and (b).

(a) Find ALL the angles that satisfy the equation

$$4 \sin^2 \theta - 3 = 0$$

over the interval $[0, 2\pi)$.

(b) Find the exact value for

$$\sin^{-1} \left(\sin \left(\frac{8\pi}{3} \right) \right).$$

A. (a) $\left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$ (b) $\frac{8\pi}{3}$

B. (a) $\left\{ \frac{\pi}{3}, \frac{2\pi}{3} \right\}$ (b) $\frac{8\pi}{3}$

C. (a) $\left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$ (b) $\frac{\pi}{3}$

D. (a) $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$ (b) $\frac{\pi}{3}$

E. (a) $\left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$ (b) $\frac{2\pi}{3}$

3. Compute the following limits:

(i) $\lim_{h \rightarrow 0} \frac{\sqrt{25 - h} - 5}{h}$

(ii) $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{x - 1}$

A. (i) DNE (ii) ∞

B. (i) $-\frac{1}{10}$ (ii) $\frac{1}{3}$

C. (i) $\frac{1}{10}$ (ii) $\frac{1}{3}$

D. (i) $-\frac{1}{10}$ (ii) DNE

E. (i) $-\frac{1}{5}$ (ii) 3

HINT: $x - 1 = (\sqrt[3]{x} - 1) \{(\sqrt[3]{x})^2 + \sqrt[3]{x} + 1\}$.

4. Compute the following limits:

(i) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x)$

(ii) $\lim_{x \rightarrow -\infty} \frac{x^3 + 2}{-2x^3 + \sqrt{4x^6 + 2}}$

A. (i) 0 (ii) ∞

B. (i) 0 (ii) DNE

C. (i) ∞ (ii) ∞

D. (i) ∞ (ii) DNE

E. (i) $\frac{1}{2}$ (ii) $-\frac{1}{4}$

5. Choose the correct answer for the number of

(a) the vertical asymptote(s)

(b) the horizontal asymptote(s)

of the following function

$$y = f(x) = \frac{x + 1}{\sqrt{x^2 + x}}$$

A. (a) 0 (b) 1

B. (a) 1 (b) 1

C. (a) 2 (b) 1

D. (a) 1 (b) 2

E. (a) 2 (b) 2

6. Compute the following limits:

(i) $\lim_{x \rightarrow (-7\pi/2)^-} \tan x$

(ii) $\lim_{x \rightarrow 1^-} \frac{x^2 - 5x + 6}{x^2 - 3x + 2}$

A. (i) $-\infty$ (ii) DNE

B. (i) DNE (ii) ∞

C. (i) ∞ (ii) $-\infty$

D. (i) $-\infty$ (ii) ∞

E. (i) ∞ (ii) ∞

7. Determine the values of the constants a and b so that the function

$$f(x) = \begin{cases} \frac{x^2 + 3x + a}{x + 1} & \text{if } x \leq -1 \\ x^3 - 3x + b & \text{if } x > -1 \end{cases}$$

becomes continuous everywhere.

- A. $a = 2, b = -1$
- B. $a = 4, b = 0$
- C. $a = 2, b = 0$
- D. $a = 4, b = 1$
- E. No values for a and b make the function continuous everywhere.

8. Suppose we have a function $f(x)$ whose derivative at $x = 3$ is equal to 5.

That is to say, we have

$$f'(3) = 5.$$

Then evaluate the following limit

$$\lim_{h \rightarrow 0} \frac{f(3 + 4h) - f(3 - 2h)}{h}.$$

A. 5

B. $\frac{5}{6}$

C. 30

D. $\frac{6}{5}$

E. We cannot evaluate the limit from the given information.

HINT: What is the value of

$$\lim_{h \rightarrow 0} \frac{f(3 + 4h) - f(3 - 2h)}{(3 + 4h) - (3 - 2h)} ?$$

9. Find the equation of the line that is tangent to the curve $y = \frac{1}{3}x^3 - 2x$ and that is also parallel to the line $y = 7x + 5$.

If there are more than one such line, then list ALL of them.

- A. $y = 7x - 18$ ONLY
- B. $y = (3x^2 - 2)x + 5$ ONLY
- C. $y = 7x - 18$ AND $y = 7x + 18$
- D. $y = 7x - 18$ AND $y = 7x$
- E. $y = 7x - 18$ AND $y = -7x + 18$

10. From the Intermediate Value Theorem, which of the following interval, can one conclude, contains a root of the equation

$$x^3 - 5x = 13 ?$$

- A. $(-1, 0)$
- B. $(0, 1)$
- C. $(1, 2)$
- D. $(2, 3)$
- E. $(3, 4)$

11. Find the exact values for the following (a) and (b)

(a) $\cos^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(-\frac{2}{3}\right)$

(b) $\sin^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(-\frac{2}{3}\right)$

A. $a = \pi, b = 0$

B. $a = 0, b = \pi$

C. $a = \pi, b = \pi$

D. $a = \pi/2, b = -\pi/2$

E. Neither $\frac{2}{3}$ nor $-\frac{2}{3}$ is one of the special values we have to memorize for the values of sine and cosine functions. Therefore, we cannot determine the value for (a) or (b).

12. Choose the right answers for (i) and (ii).

(i) Solve $\log_{10}(x^2) + 3 \log_{10} x = \log_{10} 32$ for x .

(ii) Find the value of $\cos \theta$ when $\sin \theta = -\frac{5}{13}$ and $\pi < \theta < \frac{3\pi}{2}$.

A. (i) $-\frac{3}{2} + \frac{\sqrt{137}}{2}$ (ii) $-\frac{12}{13}$

B. (i) No such x exists. (ii) $\frac{12}{13}$

C. (i) 2 (ii) $-\frac{12}{13}$

D. (i) 2 (ii) $\frac{12}{13}$

E. (i) $-\frac{3}{2} \pm \frac{\sqrt{137}}{2}$ (ii) $-\frac{5}{12}$