MA 16200
FINAL EXAM INSTRUCTIONS
VERSION 01
May 5, 2022

Your name _____

Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–25. While mark all your work on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 25 questions, each worth 8 points .The maximum possible score is 200 points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 12:25 PM, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 12:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing

   instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. Which vector is perpendicular to the plane which contains the points $P(-1,-1,1)$, $Q(1,2,3)$, and $R(3,2,-1)$?

A. $\langle 2,4,-1\rangle$

B. $\langle -2,1,1\rangle$

C. $\langle 2,-1,3\rangle$

D. $\langle -1,1,3\rangle$

E. $\langle 2,-2,1\rangle$

2. Which of the following are true and which are false?

(I) For every $\vec{u}$ and $\vec{v}$ the orthogonal vector projection of $\vec{u}$ onto $\vec{v}$, $\text{proj}_{\vec{v}}\,\vec{u}$, is equal to the orthogonal vector projection of $\vec{u}$ onto $2\vec{v}$, $\text{proj}_{2\vec{v}}\,\vec{u}$.

(II) $\vec{u} \cdot \vec{u} = |\vec{u}|^2$ for every vector $\vec{u}$.

(III) $(\vec{u} \times \vec{v}) \cdot \vec{v} = 0$ for every pair of vectors $\vec{u}$ and $\vec{v}$.

A. (I) and (III) are true; (II) is false.

B. (I) and (II) are true; (III) is false.

C. (II) and (III) are true; (I) is false.

D. All are true.

E. (II) is true; (I) and (III) false.

3. Suppose $\vec{u} \cdot \vec{v} = \sqrt{3}$ and $\vec{u} \times \vec{v} = \langle 1, 2, 2 \rangle$. What is the angle between $\vec{u}$ and $\vec{v}$?

A. $\frac{\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

E. $\frac{5\pi}{6}$

4. Find the area of the region bounded by the curves $x = 4y - y^2$ and $x = y^2 - 2y$.

A. 24

B. 9

C. 15

D. 12

E. 6

5. The region in the first quadrant bounded by the curves $y = x^2$ and $x = y$ is rotated about the axis $y = 0$. Find the volume of the resulting solid.

A. $\frac{2\pi}{15}$

B. $\frac{4\pi}{15}$

C. $\frac{2\pi}{5}$

D. $\frac{4\pi}{5}$

E. $\frac{8\pi}{5}$

6. Evaluate the integral

$$\int_0^1 x2^x dx.$$

A. $\frac{\ln 2-1}{\ln 2}$

B. $\frac{2\ln 2-1}{2\ln 2}$

C. $\frac{\ln 2-1}{(\ln 2)^2}$

D. $\frac{2\ln 2-1}{(\ln 2)^2}$

E. $\frac{\ln 2-1}{2\ln 2}$

7. Evaluate the integral

$$\int_2^3 \frac{2x+2}{x^2-x} dx.$$

A. $\ln(\frac{16}{9})$

B. $\ln(\frac{4}{9})$

C. $\ln(\frac{64}{9})$

D. $\ln(\frac{25}{9})$

E. $\ln(\frac{100}{9})$

8. Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$, $y = 0$, and $x = 1$ about the line $x = 1$. Express your answer as a definite integral.

A. $2\pi \int_0^2 (1-x)x^2 dx$

B. $2\pi \int_1^2 (1-x)x^3 dx$

C. $2\pi \int_0^1 (1-x)^2 x^3 dx$

D. $2\pi \int_1^2 (1-x)^2 x^2 dx$

E. $2\pi \int_0^1 (1-x)x^3 dx$

9. A tank of height 10 m and whose horizontal cross sections are squares of side 2 m is filled with water. How much work is required to pump all of the water out of the tank? Assume that water has density ρ kg/m^3 and that the acceleration due to gravity is g m/s^2.

A. $25\rho g$ J

B. $50\rho g$ J

C. $200\rho g$ J

D. $100\rho g$ J

E. $\frac{25}{2}\rho g$ J

10. A trigonometric substitution can be used to convert the definite integral

$$\int_2^5 \frac{dx}{\sqrt{x^2 - 4x + 13}}$$

into which of the following definite integrals?

A. $\int_0^{\pi/4} \sin(\theta)\, d\theta$

B. $\int_0^{\pi/4} \sec(\theta)\, d\theta$

C. $\int_0^{\pi/4} \cos(\theta)\, d\theta$

D. $\int_0^{\pi/4} \tan(\theta)\, d\theta$

E. $\int_0^{\pi/4} \cot(\theta)\, d\theta$

11. A swimming pool 100 m long, 10 m wide and 2 m deep is filled with water. Calculate the total force against a wall that has dimensions 10 m by 2 m. Assume that water has density ρ kg/m^3 and that the acceleration due to gravity is g m/s^2.

A. $60\rho g$ N

B. $80\rho g$ N

C. $120\rho g$ N

D. $20\rho g$ N

E. $40\rho g$ N

12. Evaluate the improper integral if it converges. If it does not converge, state that it diverges.

$$\int_0^{\pi/2} \tan(\theta)\, d\theta.$$

A. π

B. 1

C. $\pi/2$

D. The integral diverges.

E. 2π

13. Compute the surface area of the solid obtained by rotating the curve $y = \frac{1}{3}x^3$, $0 \leq x \leq 1$, about the x-axis. Express your answer as a definite integral.

A. $\frac{\pi}{3}\int_0^1 x^3\sqrt{x^4+1}\,dx$

B. $\frac{4\pi}{3}\int_0^1 x^3\sqrt{x^4+1}\,dx$

C. $\frac{2\pi}{3}\int_0^1 x^3\sqrt{x^4+1}\,dx$

D. $\frac{8\pi}{3}\int_0^1 x^3\sqrt{x^4+1}\,dx$

E. $\frac{16\pi}{3}\int_0^1 x^3\sqrt{x^4+1}\,dx$

14. Find the sum of telescopic series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

A. 1/2

B. 2/3

C. 3/4

D. 1

E. 3/2

15. Test the following series for convergence:

(I) $\sum_{n=1}^{\infty} \ln(1 + \frac{1}{n^2})$

(II) $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$

(III) $\sum_{n=1}^{\infty} (\sqrt{n^3} - \sqrt{n^3 - 1})$

A. I is convergent; II and III are divergent.

B. I and III are convergent; II is divergent.

C. I and III are divergent; II is convergent.

D. I, II and III are divergent.

E. I, II and II are convergent.

16. Represent $\int_0^1 \cos x^2 dx$ as an infinite series.

Hint: $\cos x = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{(2k)!}$

A. $\sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k)!}$

B. $\sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k+1)!}$

C. $\sum_{k=0}^{\infty}(-1)^k \frac{1}{(4k)!}$

D. $\sum_{k=0}^{\infty}(-1)^k \frac{1}{(4k+1)!}$

E. $\sum_{k=0}^{\infty}(-1)^k \frac{1}{(2k)!(4k+1)}$

17. Compute the power series that represents $\frac{1}{(1+2x)^2}$

A. $\sum_{k=0}^{\infty}(2x)^{2k}$

B. $\sum_{k=0}^{\infty} 2k(2x)^k$

C. $\sum_{k=1}^{\infty} k(-2)^{k-1}x^{k-1}$

D. $\sum_{k=1}^{\infty} k^2(2)^{-k}x^{2k-1}$

E. $\sum_{k=1}^{\infty}(-1)^k(2)^{2k-1}x^{2k+1}$

18. The first three terms of the Maclaurin series for $f(x) = (1 + 3x)^{-\frac{1}{3}}$ are

A. $1 - \frac{1}{3}x + \frac{4}{9}x^2$

B. $1 - 3x + 9x^2$

C. $1 - 2x + 4x^2$

D. $1 - \frac{1}{3}x + \frac{2}{9}x^2$

E. $1 - x + 2x^2$

19. Suppose we want to approximate the sum of the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}$ by the sum $s_m = \sum_{k=1}^{m} \frac{(-1)^k}{k^3}$ of the first m terms. By the theory of alternating series, the error will be less than or equal to 10^{-3} provided $m =$

A. 4

B. 5

C. 6

D. 7

E. 9

20. The equation $4\cos\theta + \sin\theta = \frac{2}{r}$ in polar coordinates represents a part of

A. a straight line

B. a circle

C. a parabola

D. an ellipse which is not a circle

E. a cycloid

21. What is the length of arc of the spiral given in polar coordinates by

$$r = e^{-2\theta}, 0 \leq \theta \leq 1/2?$$

A. $\frac{1+e}{\sqrt{6}}$

B. $\frac{1-e}{2\sqrt{3}}$

C. $4(1 - e^{-1})$

D. $e^2 - 1$

E. $\frac{(e-1)\sqrt{5}}{2e}$

22. $\sum_{n=1}^{\infty} \frac{3+2^n}{2^{2n}} =$

A. 1

B. 4/3

C. 5/2

D. 3/2

E. 2

23. For which of the following series the ratio test is inconclusive?

(I) $\sum_{n=1}^{\infty} \frac{7^n}{n!}$

(II) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(III) $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

A. I only

B. II only

C. III only

D. I and II

E. II and III

24. The area between the spirals $r = \theta$ and $r = 2\theta$, $0 \leq \theta \leq \pi/2$ is

A. $\pi^2/4$

B. $\pi^2 - 2$

C. $\pi^3/16$

D. $\pi^3 + 4\pi$

E. $\pi^4/8$

25. Which is true? The point whose Cartesian coordinates are $(1, \sqrt{3})$ has polar coordinates

I. $(2, \pi/3)$ II. $(2, -5\pi/3)$ III. $(2, 4\pi/3)$

A. Only I.

B. Only II.

C. Only III.

D. Only I and II.

E. All are true.