MA 16200
FINAL EXAM Green
May 2, 2019

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Write the following in the TEST/QUIZ NUMBER boxes: **00** (and blacken in the appropriate digits below the boxes)

You must use a #2 pencil on the mark–sense sheet (answer sheet).On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **25** questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the mark–sense sheet and the exam booklet when you are finished.

If you finish the exam before 9:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 9:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. Find a vector that has the same direction as $\langle -2, 6, -3 \rangle$ but has length 3.

A. $\langle -\frac{6}{7}, \frac{18}{7}, -\frac{9}{7} \rangle$

B. $\langle -6, 18, -9 \rangle$

C. $\langle -\frac{2}{\sqrt{3}}, 2\sqrt{3}, -\sqrt{3} \rangle$

D. $\langle -\frac{2}{3}, 2, -1 \rangle$

E. $\langle -\frac{2}{7}, \frac{6}{7}, -\frac{3}{7} \rangle$

2. Suppose $\vec{a} \cdot \vec{b} = 5$ and $\vec{a} \times \vec{b} = \vec{\imath} + 5\vec{\jmath} + 7\vec{k}$. What is the angle between $\vec{a}$ and $\vec{b}$?

A. $\dfrac{\pi}{4}$

B. $\dfrac{2\pi}{3}$

C. $\dfrac{\pi}{6}$

D. $\dfrac{\pi}{3}$

E. $\dfrac{\pi}{2}$

3. Find the area of the region bounded by the curve $y = xe^{-x}$ and the x-axis from $x = 0$ to $x = 4$.

A. $-5e^{-4}$

B. $4e^{-4}$

C. $1 - 5e^{-4}$

D. $1 - 3e^{-4}$

E. $3e^{-4}$

4. $\displaystyle\int x \tan^2 x \, dx$

A. $\sec^2 x - \tan x - \dfrac{x^2}{2} + C$

B. $x \tan x + \ln|\cos x| - \dfrac{x^2}{2} + C$

C. $x \sec^2 x + \ln|\sec x| - \dfrac{x^2}{2} + C$

D. $x \sec^2 x - \tan x + C$

E. $x \tan x + \ln|\sec x + \tan x| + C$

5. $\displaystyle\int \frac{dx}{x(x^2+1)^2}$

A. $\ln\left(\dfrac{|x|}{\sqrt{x^2+1}}\right)+\dfrac{1}{2(x^2+1)}+C$

B. $\ln\left(\dfrac{|x|}{\sqrt{x^2+1}}\right)+\dfrac{1}{(x^2+1)}+C$

C. $\ln\left(\dfrac{1}{\sqrt{x^2+1}}\right)+\dfrac{1}{2(x^2+1)}+C$

D. $\ln\left(\dfrac{1}{\sqrt{x^2+1}}\right)+\dfrac{1}{(x^2+1)}+C$

E. $\ln\left(\dfrac{|x|}{\sqrt{x^2+1}}\right)+C$

6. Find the volume of the solid generated by revolving about the x-axis the region bounded by $y=\sin x\cos x$ and the x-axis, between $x=0$ and $x=\pi/2$.

A. $\dfrac{\pi^2}{16}$

B. $\dfrac{\pi^2}{2}$

C. $\dfrac{\pi^2}{4}$

D. $\dfrac{\pi^2}{32}$

E. $\dfrac{\pi^2}{8}$

7. What is the **average value** of the function $f(x) = \dfrac{1}{(1-x^2)^{3/2}}$ over the interval $0 \leq x \leq \dfrac{3}{5}$?

A. $\dfrac{4}{5}$

B. $\dfrac{5}{4}$

C. 1

D. $\dfrac{4}{3}$

E. $\dfrac{3}{4}$

8. Evaluate $\displaystyle\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$

A. $\dfrac{\pi}{12}$

B. $\dfrac{\pi}{3}$

C. $\dfrac{\pi}{8}$

D. $\dfrac{\pi}{4}$

E. $\dfrac{\pi}{6}$

9. Evaluate $\displaystyle\int_1^3 \frac{dx}{(x-1)^{2/3}}$

A. $3\sqrt{3}$

B. $3\sqrt{2}$

C. This improper integral diverges.

D. $3\sqrt[3]{3}$

E. $3\sqrt[3]{2}$

10. Find the area of the surface obtained by rotating the curve $y = \dfrac{x^3}{3}$ about the x-axis, for $0 \le x \le 1$.

A. $\dfrac{\pi\sqrt{2}}{6} - \dfrac{\pi}{6}$

B. $\dfrac{\pi \cdot 2\sqrt{2}}{9} - \dfrac{\pi}{9}$

C. $\dfrac{\pi\sqrt{2}}{6} - \dfrac{\pi}{12}$

D. $\dfrac{\pi}{12}$

E. $\dfrac{\pi}{9}$

11. Let $a_n = \dfrac{\cos(1/n)}{2n+1}$. Which one of the following is true?

A. The sequence $\{a_n\}$ is divergent and the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is divergent.

B. The sequence $\{a_n\}$ is convergent and the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is divergent.

C. The sequence $\{a_n\}$ is divergent and the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

D. The sequence $\{a_n\}$ is convergent and the series $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

E. None of these statements are true.

12. Which statement is true about these three series?

$$(i)\ \sum_{k=1}^{\infty} k^2 e^{-k^3} \quad , \quad (ii)\ \sum_{\ell=2}^{\infty} \frac{1}{\sqrt{\ell - 1/\ell}} \quad , \quad (iii)\ \sum_{m=1}^{\infty} \frac{m!}{3^m}$$

A. (*ii*) converges. The others diverge.

B. All three diverge.

C. (*iii*) converges. The others diverge.

D. (*i*) converges. The others diverge.

E. Two of them converge and one diverges.

13. Which statement is true about these two series?

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \quad , \quad \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

A. Both are conditionally convergent.

B. One is conditionally convergent and one is divergent.

C. One is conditionally convergent and one is absolutely convergent.

D. One is divergent and one is absolutely convergent.

E. Both are absolutely convergent.

14. Only one of these series diverges. Which one?

A. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{(1.01)^n}{n+3^n}$

D. $\displaystyle\sum_{n=1}^{\infty} \tan(1/n)$

E. $\displaystyle\sum_{n=1}^{\infty} \left(e^{1/n}-1\right)^n$

15. Find the sum.

$$1 + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{3^2} + 4 \cdot \frac{1}{3^3} + 5 \cdot \frac{1}{3^4} + \dots$$

Hint: What if the $\frac{1}{3}$'s were x's?

A. 3

B. 2

C. $\frac{7}{3}$

D. $\frac{9}{4}$

E. $\frac{3}{2}$

16. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} n^n x^n$

A. $(-1, 1)$

B. $(-1, 1]$

C. $[-1, 1)$

D. $\{0\}$

E. $[-1, 1]$

17. Find the first few terms of the Taylor series centered at $a = 2$ for the function $f(x) = 1/x$.

A. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{4}(x-2)^2 - \frac{3}{8}(x-2)^3 + \dots$

B. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 + \dots$

C. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{12}(x-2)^2 - \frac{1}{48}(x-2)^3 + \dots$

D. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{16}(x-2)^2 - \frac{1}{96}(x-2)^3 + \dots$

E. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{6}(x-2)^2 - \frac{1}{8}(x-2)^3 + \dots$

18. Find $\lim_{x\to 0} \dfrac{x^6 - 12x^2 + 24\tan^{-1}(x^2/2)}{x^{10}}$

Hint: $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1}$

A. $\frac{3}{16}$

B. $\frac{3}{20}$

C. The limit does not exist.

D. $\frac{3}{10}$

E. 0

19. Find the length of the curve $x = e^t \cos t,\ y = e^t \sin t$ for $0 \leq t \leq \ln 2$.

A. $\sqrt{2}$

B. $\dfrac{\sqrt{2}}{2}$

C. $\dfrac{\sqrt{3}}{2}$

D. $2\sqrt{2}$

E. $\dfrac{3}{2}$

20. Find a Cartesian equation for the curve $r = \tan\theta \sec\theta$

A. $y = \tan x$

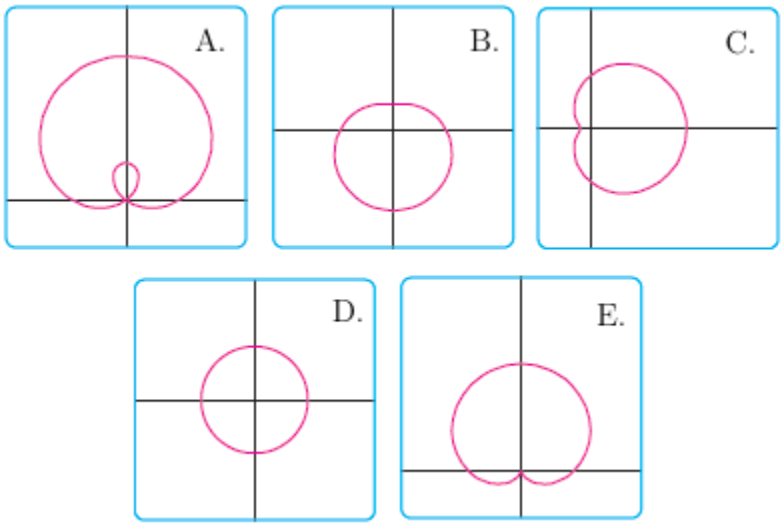
B. $\sqrt{x^2 + y^2} = \dfrac{y}{x^2}$

C. $x = y^2$

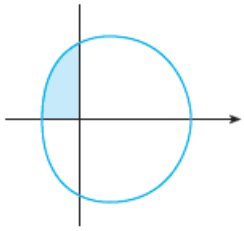
D. $y = x^2$

E. $y = \sqrt{1 + x^2}$

21. Find the graph of $r = 1 + \sin\theta$



22. The curve pictured below is $r = 2 + \cos\theta$. Find the area of the shaded region:



A. $\dfrac{9\pi}{8} - 2$

B. $\dfrac{9\pi}{8}$

C. $\dfrac{9\pi}{4} - 2$

D. $\pi - 1$

E. $\dfrac{9\pi}{4}$

23. Find the length of the polar curve $r = \theta^2$, where $0 \leq \theta \leq \pi$.

A. $\dfrac{\pi^{3/2}}{3}$

B. $\dfrac{\pi^3}{3}$

C. $(\pi^2 + 4)^{3/2}$

D. $\dfrac{(\pi^2 + 2)^{3/2} - 2\sqrt{2}}{3}$

E. $\dfrac{(\pi^2 + 4)^{3/2} - 8}{3}$

24. Find one of the foci of the hyperbola

$$3x^2 - y^2 - 18x - 6y + 9 = 0$$

A. $(3, -3)$

B. $(2\sqrt{3}, -3)$

C. $(\sqrt{3}, -3)$

D. $(3 + 2\sqrt{3}, -3)$

E. $(3 + \sqrt{3}, -3)$

25. Four of these complex numbers are equal to each other. Which one is not?

A. $2e^{2\pi i/3}$

B. $\dfrac{4}{-1-\sqrt{3}i}$

C. $-1+\sqrt{3}i$

D. $2\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)$

E. $2e^{4\pi i/3}$