MA 16200
FINAL EXAM Form A
May 5, 2016

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the mark–sense sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets.

9. Turn in both the mark–sense sheets and the question sheets when you are finished.

10. If you finish the exam before 2:50 pm, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 2:50 pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. Find the cosine of the angle between $\mathbf{v} = \langle 1, 2, 1 \rangle$ and $\mathbf{w} = \langle -1, 1, 1 \rangle$.

A. $\dfrac{2}{\sqrt{3}}$

B. $\dfrac{\sqrt{2}}{3}$

C. $\dfrac{\sqrt{6}}{2}$

D. $\sqrt{3}$

E. $\dfrac{\sqrt{3}}{2}$

2. Find the area of the triangle with vertices

$$(1, 0, -2), \quad (2, 1, -1), \quad (3, 2, -1).$$

A. $\dfrac{\sqrt{3}}{2}$

B. $\dfrac{\sqrt{5}}{2}$

C. $2\sqrt{2}$

D. $\dfrac{\sqrt{2}}{2}$

E. $\sqrt{6}$

3. Use the disk/washer method to find the volume of the solid obtained by revolving the region bounded by $y = x^2$ and $y = 1$ about the line $y = 1$

A. $\frac{8}{15}\pi$

B. $\frac{8}{5}\pi$

C. $\frac{16}{5}\pi$

D. $\frac{16}{15}\pi$

E. $\frac{2}{3}\pi$

4. The region bounded by $y = x^2$ and $y = 4$ is revolved about the line $x = 2$. If the method of cylindrical shells is used, the integral that represents the volume of the solid is

A. $4\pi \int_0^2 (4 - x)(2 - x^2)\, dx$

B. $2\pi \int_{-2}^2 (2 - x)(4 - x^2)\, dx$

C. $2\pi \int_{-2}^2 x(4 - x^2)\, dx$

D. $4\pi \int_0^4 \sqrt{y}\, dy$

E. $8\pi \int_0^4 y\, dy$

5. Find the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0, 2]$

A. $\dfrac{26}{9}$

B. $\dfrac{52}{9}$

C. $\dfrac{26}{3}$

D. $\dfrac{52}{3}$

E. 24

6. Find the integral $\displaystyle\int e^{2x}\sin x\,dx$

A. $\dfrac{e^{2x}}{5}(\sin x - \cos x) + C$

B. $\dfrac{e^{2x}}{5}(\sin x + \cos x) + C$

C. $\dfrac{e^{2x}}{5}(2\sin x + \cos x) + C$

D. $\dfrac{e^{2x}}{5}(\sin x + 2\cos x) + C$

E. $\dfrac{e^{2x}}{5}(2\sin x - \cos x) + C$

7. Find the integral: $\displaystyle\int_0^{\pi/2} \cos^5 x \, dx$

A. $\dfrac{1}{15}$

B. $\dfrac{2}{15}$

C. $\dfrac{4}{15}$

D. $\dfrac{7}{15}$

E. $\dfrac{8}{15}$

8. Which trigonometric substitution should be used to compute

$$\int \frac{x^2}{\sqrt{4-9x^2}}\, dx \qquad ?$$

A. $x = 2\tan\theta$

B. $x = \dfrac{2}{3}\tan\theta$

C. $x = 2\sin\theta$

D. $x = \dfrac{2}{3}\sin\theta$

E. $x = \dfrac{2}{3}\sec\theta$

9. Compute

$$\int_2^3 \frac{(x+3)}{x^2-1}\,dx$$

A. $2\ln 2 + \ln 2$

B. $\ln 2 + \ln 3$

C. $2\ln 2$

D. $\ln 3$

E. $3\ln 2 + \ln 3$

10. Evaluate

$$\int_0^\infty \frac{dx}{(2x+3)^3}$$

A. $\dfrac{1}{36}$

B. $\dfrac{1}{12}$

C. $\dfrac{1}{18}$

D. $\dfrac{1}{9}$

E. $\dfrac{2}{9}$

11. Use the formula

$$\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$$

to compute $\displaystyle\int_2^3 \frac{dx}{(x^2-4x+7)^{3/2}}$

A. $\dfrac{1}{2}$

B. $\dfrac{1}{2\sqrt{3}}$

C. $\dfrac{1}{6}$

D. $\dfrac{1}{3}$

E. $\dfrac{1}{\sqrt{3}}$

12. Which of the following integrals gives the length of the graph of $y = \sqrt{x}$ on the interval $[1, 9]$?

A. $\displaystyle\int_1^9 \sqrt{x^2+x}\, dx$

B. $\displaystyle\int_1^9 \sqrt{x+\sqrt{x}}\, dx$

C. $\displaystyle\int_1^9 \sqrt{x+\frac{1}{2\sqrt{x}}}\, dx$

D. $\displaystyle\int_1^9 \sqrt{1+\frac{1}{2\sqrt{x}}}\, dx$

E. $\displaystyle\int_1^9 \sqrt{1+\frac{1}{4x}}\, dx$

13. Which of the following **sequences** converge?

I. $\left\{\dfrac{1}{n}\right\}$

II. $\left\{\dfrac{(-1)^{n+1}}{n}\right\}$

III. $\left\{\dfrac{n}{\ln n}\right\}$

IV. $\left\{\dfrac{n^2}{e^n}\right\}$

A. None of them

B. II, IV

C. I, II, IV

D. II, III, IV

E. All of them

14. The sum of the series $2 - 1 + \dfrac{1}{2} - \dfrac{1}{4} + \dfrac{1}{8} - \cdots$ is

A. 4/3

B. 5/4

C. 1

D. 3/2

E. 3/4

15. Given the two series $A = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$ and $B = \sum_{n=1}^{\infty} n^5 e^{-n^6}$, use the Integral Test to determine whether each series is convergent and choose the correct statement from below:

A. Both series are divergent

B. Series A is convergent, series B is divergent

C. Series A is divergent, series B is convergent

D. Both series are convergent

E. The Integral Test is inconclusive

16. **How many** of the following series are convergent?

$$\sum_{n=2}^{\infty} (-1)^n \sqrt{n}$$

$$\sum_{n=2}^{\infty} \frac{n^3}{n^4 + 5}$$

$$\sum_{n=1}^{\infty} \frac{n!}{n^{2002}}$$

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n}$$

A. One

B. Two

C. Three

D. Four

E. Five

17. **How many** of the following series are **conditionally** convergent?

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{0.998}}$$

$$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.001}}$$

A. One

B. Two

C. Three

D. Four

E. Five

18. If the power series of $\ln(1+x^2)$ is used to express $S = \displaystyle\int_0^1 \ln(1+x^2)\, dx$ as an infinite series, what is the maximum possible error when the first three terms of that series are used to approximate S? Note that $\ln(1+x) = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

A. $\dfrac{1}{21}$

B. $\dfrac{1}{20}$

C. $\dfrac{1}{36}$

D. $\dfrac{1}{18}$

E. $\dfrac{1}{12}$

19. Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n (x-2)^n}{n+1}$

A. $(0, 4)$

B. $\left(\frac{3}{2}, \frac{5}{2}\right]$

C. $(0, 4]$

D. $\left[\frac{3}{2}, \frac{5}{2}\right)$

E. $\left(\frac{3}{2}, \frac{5}{2}\right)$

20. Suppose that the function $f(x) = x^4 - x^3 - x + 2$ is written as a Taylor series about $a = 2$, what is the coefficient of $(x-2)^3$?

A. 3

B. 5

C. 4

D. 6

E. 7

21. What is the coefficient of x^3 in the Maclaurin series of $\dfrac{1}{(x-2)^2}$?

A. $\dfrac{1}{8}$

B. $\dfrac{3}{8}$

C. $\dfrac{5}{8}$

D. $\dfrac{7}{8}$

E. $\dfrac{9}{8}$

22. One of these parametric curves goes twice around a circle in the clockwise direction. Which one?

A. $x = 2\cos t + 3, \quad y = 2\sin t - 1, \quad 0 \le t \le 4\pi$

B. $x = \cos(2t) + 3, \quad y = -\sin(2t) - 1, \quad 0 \le t \le 4\pi$

C. $x = \sin(2t) + 3, \quad y = \cos(2t) - 1, \quad 0 \le t \le 4\pi$

D. $x = 2\sin t + 3, \quad y = -2\cos t - 1, \quad 0 \le t \le 4\pi$

E. $x = 2\cos t + 3, \quad y = -2\sin t - 1, \quad 0 \le t \le 4\pi$

23. For the parametric curve

$$x = 2\sin t, \quad y = 3\cos t, \quad -\pi \leq t \leq \pi,$$

what is $\dfrac{d^2y}{dx^2}$ when $t = 0$?

A. -12

B. -6

C. -3

D. $-\dfrac{3}{2}$

E. $-\dfrac{3}{4}$

24. If $(2 + 3i)e^{i\pi/4} = a + bi$, what is b?

A. $\dfrac{\sqrt{2}}{2}$

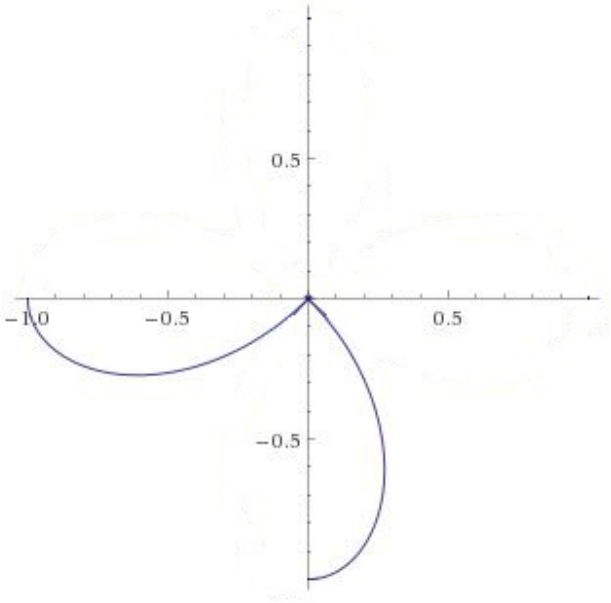
B. $\dfrac{3\sqrt{2}}{2}$

C. $\dfrac{5\sqrt{2}}{2}$

D. $\dfrac{7\sqrt{2}}{2}$

E. $\dfrac{9\sqrt{2}}{2}$

25. The picture below shows two pieces of the polar curve $r = \cos 2\theta$. Which θ intervals correspond to these two pieces?



A. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right], \left[\frac{3\pi}{4}, \pi\right]$

B. $\left[\frac{3\pi}{4}, \pi\right], \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right]$

C. $\left[\pi, \frac{5\pi}{4}\right], \left[\frac{3\pi}{2}, \frac{7\pi}{4}\right]$

D. $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right], \left[\pi, \frac{5\pi}{4}\right]$

E. $\left[\frac{5\pi}{4}, \frac{3\pi}{2}\right], \left[\frac{7\pi}{4}, 2\pi\right]$