MA 162 Final Exam Spring, 2003

NAME _____

STUDENT ID _____

REC. INSTR. _____ REC. TIME _____

INSTRUCTIONS:

1. Verify that you have all the pages (there are 14 pages including this cover page).
2. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on the mark–sense answer sheet, and fill in the corresponding circles.
3. Mark the letter of your response for each question on the mark–sense answer sheet.
4. There are 25 problems. Credit for each problem is given in the parenthesis to the left of each problem number.
5. No books or notes or calculators may be used.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ |x| < \infty \qquad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1},\ |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n},\ |x| < \infty \qquad (1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n,\ |x| < 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n},\ |x| < 1 \qquad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n,\ |x| < 1$$

$$\sin^2 x = \frac{1-\cos(2x)}{2} \qquad \cos^2 x = \frac{1+\cos(2x)}{2}$$

(4 pts) 1. If $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ are vectors then $(\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \times \mathbf{d})$ is a vector.

A. True

B. False

(8 pts) 2. Compute $(-2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} - 3\mathbf{j} + \mathbf{k})$

A. $\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

B. $-2\mathbf{i} - 4\mathbf{j} - 10\mathbf{k}$

C. $2\mathbf{i} - 4\mathbf{j} + 10\mathbf{k}$

D. $2\mathbf{i} + 4\mathbf{j} + 10\mathbf{k}$

E. $-\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$

(9 pts) 3. $\displaystyle\int_4^\infty \frac{1}{(x-1)^2}\,dx =$

A. $\frac{1}{27}$

B. $\frac{1}{9}$

C. $\frac{1}{3}$

D. ∞

E. $-\infty$

(7 pts) 4. The length of the curve $y = \ln(\sec x), \quad 0 \le x \le \frac{\pi}{4}$ is

A. $\sqrt{2}$

B. $\ln\sqrt{2}$

C. $\ln\left(\frac{\sqrt{3}}{2}\right)$

D. $\ln\left(1+\frac{\sqrt{3}}{2}\right)$

E. $\ln(1+\sqrt{2})$

(4 pts) 5. $\int_0^{\pi} (\cos(x) - \cos(2x))\, dx$ represents the area between $y = \cos(x)$ and $y = \cos(2x)$ between $x = 0$ and $x = \pi$.

A. True

B. False

(10 pts) 6. The region enclosed by the curves $y = x$ and $y = x^2$ is rotated about the y-axis. The volume of the resulting solid is

A. $\frac{6}{55}\pi$

B. $\frac{1}{12}\pi$

C. $\frac{1}{6}\pi$

D. $\frac{1}{3}\pi$

E. $\frac{2}{15}\pi$

(10 pts) 7. Compute $\displaystyle\int_1^4 \frac{\ln x}{2\sqrt{x}}\,dx$.

A. $2\ln 4 + 2$

B. $\ln 16 - 2$

C. $2(\ln 4 + 2)$

D. $\ln 4 - 2$

E. $\ln 16 + 2$

(11 pts) 8. Compute $\displaystyle\int_0^{\pi/4} \tan^3 x \sec x\,dx$.

A. $\dfrac{\sqrt{2}-2}{3}$

B. $\dfrac{\sqrt{2}+2}{3}$

C. $\sqrt{2}+\dfrac{2}{3}$

D. $\dfrac{2}{3}-\sqrt{2}$

E. $\dfrac{2-\sqrt{2}}{3}$

(7 pts) 9. Which of the following substitutions should be used to find $\int x^5\sqrt{9-4x^2}\,dx$.

A. $x = 3\sin\theta$

B. $x = \frac{2}{3}\sin\theta$

C. $x = \frac{2}{3}\sec\theta$

D. $x = \frac{3}{2}\sin\theta$

E. $x = \frac{3}{2}\sec\theta$

(8 pts) 10. Find $\int_6^7 \frac{2(x-1)}{(x-5)(x+3)}\,dx$.

A. $\ln(20/9)$

B. $\ln(10/9)$

C. $\ln(5/3)$

D. $\ln(9/5)$

E. $\ln(9/10)$

(8 pts) 11. Determine whether the series $1 - \frac{4}{3} + \frac{16}{9} - \frac{64}{27} + \cdots$ is convergent or divergent. If it is convergent, find its sum.

A. $\frac{3}{7}$

B. $\frac{4}{7}$

C. $\frac{4}{3}$

D. $\frac{7}{3}$

E. divergent

(10 pts) 12. The series $\sum_{n=2}^{\infty} \frac{(x+1)^n}{n^2 2^n}$ converges for

A. $-3 \leq x < 1$

B. $-3 \leq x \leq 1$

C. $-1 < x < 3$

D. $-1 < x \leq 3$

E. $-\infty < x < \infty$

(10 pts) 13. Find a power series representation for the function $f(x) = \dfrac{x^2}{1-x^2}$.

A. $-1 + x^2 - x^4 + x^6 - \cdots$

B. $1 - x^2 + x^4 - x^6 + \cdots$

C. $-x^2 - x^4 - x^6 - x^8 - \cdots$

D. $x^2 + x^4 + x^6 + x^8 + \cdots$

E. $1 + x^2 + x^4 + x^6 + \cdots$

(4 pts) 14. If $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n2^n}$, then $f^{(10)}(0) =$

A. $\dfrac{1}{(10)2^{10}}$

B. $\dfrac{9!}{2^{10}}$

C. 0

D. $\dfrac{10!}{2^{10}}$

E. $\dfrac{1}{2^{10}9!}$

(4 pts) 15. If $\lim_{n\to\infty} n^3 a_n = 5$ then $\sum_{n=1}^{\infty} a_n$ is convergent.

A. True

B. False

(6 pts) 16. The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{3n}$ is

A. Absolutely convergent

B. Conditionally convergent

C. Divergent

(6 pts) 17. The series $\sum_{n=0}^{\infty}(-1)^{n+1}\left(\frac{9+3n}{7n}\right)^n$ is

A. Absolutely convergent

B. Conditionally convergent

C. Divergent

(10 pts) 18. Find the first three terms of Taylor series of $\frac{1}{(3+x)^2}$ at 0.

A. $\frac{1}{3}-\frac{2}{9}x+\frac{1}{9}x^2$

B. $\frac{1}{9}-\frac{1}{9}x+\frac{2}{9}x^2$

C. $1-\frac{2}{3}x+\frac{1}{3}x^2$

D. $\frac{1}{9}-\frac{2}{27}x+\frac{1}{27}x^2$

E. $\frac{1}{9}-\frac{1}{27}x+\frac{2}{27}x^2$

(9 pts) 19. The tangent line to the curve $x = t^3 - 2t^2 - 1$, $y = 1 - 2t^2$ when $t = 1$ is

A. $y = -4x - 9$

B. $y = 4x + 7$

C. $y = x$

D. $y = -4x + 7$

E. $y = 4x$

(10 pts) 20. Which of the following represents the arc length of the curve $x = e^{2t}$, $y = \sin 4t$, $0 \le t \le \pi$?

A. $\displaystyle\int_0^\pi \sqrt{e^{4t} + \sin^2(4t)}\, dt$

B. $\displaystyle\int_0^\pi \sqrt{e^{4t} + 16\cos^2(4t)}\, dt$

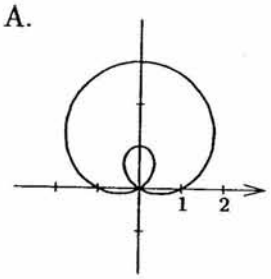
C. $\displaystyle\int_0^\pi \sqrt{4e^{4t} + \cos^2(4t)}\, dt$

D. $\displaystyle\int_0^\pi 2\sqrt{e^{4t} + 4\cos^2(4t)}\, dt$

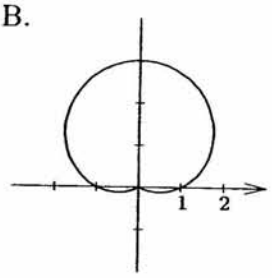
E. $\displaystyle\int_0^\pi \sqrt{4e^{4t} + 4\sin^2(4t)}\, dt$

(9 pts) 21. Which of the following is the graph of the polar curve $r = 2\sin\theta - 1$, $0 \le \theta \le \pi$.

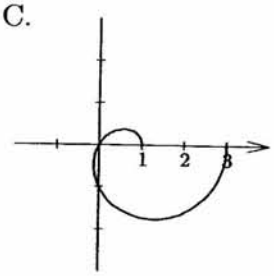
A.



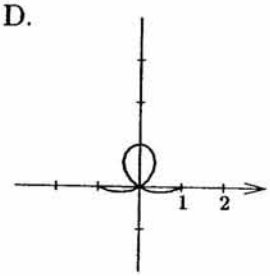
B.



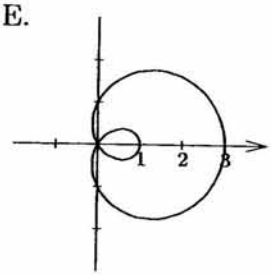
C.



D.



E.



(10 pts) 22. The length of the polar curve $r = 2 + \cos\theta$, $\quad 0 \le \theta \le \pi$ is

A. $2\displaystyle\int_0^{\pi/2} \sqrt{5 + 4\cos\theta}\, d\theta$

B. $\displaystyle\int_0^{\pi} \sqrt{5 + 4\cos\theta}\, d\theta$

C. $2\displaystyle\int_0^{\pi/2} \sqrt{2 + \cos\theta}\, d\theta$

D. $\displaystyle\int_0^{\pi} \sqrt{2 + \cos\theta}\, d\theta$

E. $\displaystyle\int_0^{\pi} \sqrt{2 + \cos\theta - \sin\theta}\, d\theta$

(11 pts) 23. The area of the region enclosed by one loop of the curve $r = 4\sin(3\theta)$ is

A. $\frac{4}{3}\pi$

B. $\frac{3}{4}\pi$

C. $\frac{2}{3}\pi$

D. $\frac{3}{2}\pi$

E. $\frac{1}{3}\pi$

(8 pts) 24. Find an equation of the ellipse with foci $(0,0)$ and $(4,0)$ and vertex $(-1,0)$.

A. $\frac{x^2}{5} + \frac{(y-2)^2}{9} = 1$

B. $\frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$

C. $\frac{x^2}{9} + \frac{(y-2)^2}{5} = 1$

D. $\frac{(x+2)^2}{5} + \frac{y^2}{9} = 1$

E. $\frac{(x-2)^2}{5} + \frac{y^2}{9} = 1$

(7 pts) 25. A polar equation of a hyperbola with a focus at the origin, eccentricity 2 and directrix $x = -2$ is

A. $r = \dfrac{4}{1 - 2\cos\theta}$

B. $r = \dfrac{4}{1 + 2\cos\theta}$

C. $r = \dfrac{4}{1 - 2\sin\theta}$

D. $r = \dfrac{4}{1 + 2\sin\theta}$

E. $r = \dfrac{2}{1 - 2\cos\theta}$