MA 16200
FINAL EXAM Form 01
December 15, 2022

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below the boxes on the scantron: **01**

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough space for your name, fill in as much as you can.
2. Section number. If you don't know your section number, ask your TA.
3. Test/Quiz number: **01**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros.**

There are **25** questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 7:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: _____

1. Find an equation for a sphere with center $(0, -8, -5)$ and radius 8.

A. $x^2 + y^2 + z^2 + 16y + 10z = -25$
B. $x^2 + y^2 + z^2 + 16y - 10z = -25$
C. $x^2 + y^2 + z^2 - 16y + 10z = -25$
D. $x^2 + y^2 + 8z^2 + 16y - 10z = -25$
E. $x^2 + y^2 + 8z^2 + 16y + 10z = -25$

2. If $\vec{v} =< x, 2 >$ and $\vec{w} =< 1, x >$, find a positive x such that $|\vec{v} + \vec{w}| = \sqrt{61}$

A. 4
B. 5
C. 6
D. 7
E. 8

3. Find the volume of the solid obtained by revolving the region in the first quadrant bounded by $y = \dfrac{2}{x}$, $x = 2$, and $y = 2$ about the $y-$axis.

A. 4π

B. $\frac{4}{3}\pi$

C. 6π

D. 2π

E. $\frac{1}{2}\pi$

4. Find a vector that is perpendicular to the plane which contains the points $P(1, 2, 3)$, $Q(3, 1, -1)$, and $R(3, 3, 1)$.

A. $< 4, -4, 1 >$

B. $< 1, 1, 1 >$

C. $< 4, -4, -1 >$

D. $< -2, -4, 4 >$

E. $< 1, -1, 1 >$

5. Find the volume of the solid obtained by revolving the region bounded by $y = e^{x/2} + 1$, $y = 3$, $x = 0$, and $x = 1$ about the line $y = 1$.

A. $\pi(5 - e)$

B. $\pi(e^2 - 4)$

C. $\pi(e - 4)$

D. $2\pi(e - 4)$

E. $\pi(7 - e)$

6. A 10−m chain hangs from a wench. If the chain weighs 1.5 kg/m, find the work in J to wind up the entire chain. Use $g = 10$ m/s^2 in your calculation.

A. 500

B. 750

C. 1000

D. 1250

E. 1500

7. What is the most appropriate substitution or procedure to compute

$$\int \sin^6 x \cos^7 x \, dx$$

A. substitution with $u = \cos x$

B. substitution with $u = \sin x$

C. substitution with $u = \sin^4 x$

D. substitution with $u = \cos^5 x$

E. use the identities $\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$

8. $\int_0^1 xe^{3x} \, dx =$

A. $\frac{2}{9}e^3$

B. $\frac{1}{9} + \frac{2}{9}e^3$

C. 1

D. $\frac{1}{9}$

E. $\frac{1}{9}e^3 - 1$

9. What is the most appropriate substitution to compute

$$\int_0^{\frac{\pi}{4}} \sec^6 x \, dx$$

A. $u = \tan x$

B. $u = \sec x$

C. $u = \tan^2 x$

D. $u = \sec^2 x$

E. $u = \cos^2 x$

10. To compute $\displaystyle\int \frac{2x^3 + 1}{x^2 - 4x + 4}\, dx$, one should reduce the integrand to

A. $2x + \frac{A}{x^2} + \frac{B}{x} + C$

B. $2x + 8 + \frac{A}{x-2} + \frac{B}{x-2}$

C. $2 + \frac{A}{x-2} + \frac{B}{(x-2)^2}$

D. $2x + \frac{A}{x-2} + \frac{B}{(x-2)^2}$

E. $2x + 8 + \frac{A}{x-2} + \frac{B}{(x-2)^2}$

11. $\displaystyle\int \frac{1}{x^2\sqrt{4x^2+1}}\,dx =$

A. $-\sqrt{4x^2+1}+C$

B. $-\frac{\sqrt{4x^2+1}}{x}+C$

C. $-2\sin^{-1}x+C$

D. $-\frac{2}{x}+C$

E. $-\frac{x}{\sqrt{4x^2+1}}+C$

12. To compute $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^2-2x+2}}\,dx$, the first step is to transform the integral to:

A. $\displaystyle\int_0^{\pi/2} \sin^3 t\tan t\,dt$

B. $\displaystyle\int_{-\pi/4}^{0} (1+\sin t)^3\tan t\,dt$

C. $\displaystyle\int_{-\pi/4}^{0} (1+\tan t)^3\sec t\,dt$

D. $\displaystyle\int_0^{\pi/2} \sin^3 t(1+\sec t)\,dt$

E. $\displaystyle\int_0^{\pi/4} \tan^3 t\sec t\,dt$

13. $\displaystyle\int_1^{\infty} \frac{3t+1}{t^2}\,dt =$

A. 1

B. 2

C. 1/2

D. $\ln 2$

E. The integral is divergent

14.

$$I.\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}-3}{n}; \quad II.\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\ln^2(n+1)}; \quad III.\ \sum_{n=1}^{\infty}(-1)^{n-1}\sin(n).$$

Which of the following statement is true?

A. All three are convergent

B. Only II is convergent

C. II and III are convergent

D. All three are divergent

E. I and II are convergent

15. Consider $S = \sum_{m=1}^{\infty} (-1)^{m-1} \frac{1}{m^3}$ and its partial sum S_n. Determine the least number n such that $|S - S_n| \leq 8 \times 10^{-6}$.

A. $n = 101$

B. $n = 49$

C. $n = 99$

D. $n = 51$

E. $n = 999$

16. Determine which of the following statements are true and which are false.

(I) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} |a_n|$ is convergent.

(II) If $a_n > 0$, $b_n > 0$, $\sum_{n=1}^{\infty} b_n$ is convergent, and $b_n \leq a_n$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

(III) If $a_n > 0$, $b_n > 0$, $\sum_{n=1}^{\infty} b_n$ is convergent, and $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{1}{2}$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

A. II is true; I and III are false.

B. III is true; I and II are false.

C. I is true; II and III are false.

D. I and II are true; III is false.

E. I, II, and III are false.

17. Determine if the series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

A. Converges to 3/2.

B. Converges to 33/6.

C. Converges to 11/6.

D. Converges to 9/2.

E. Diverges.

18. Find the interval of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{(x-9)^k}{9+7k}$$

A. $(8, 10)$

B. $[8, 10)$

C. $(8, 10]$

D. $[8, 10]$

E. $(-\infty, \infty)$

19. What is the fourth term of the Taylor series representation of the function $f(x) = \ln x$ centered at $a = 2$?

A. $\frac{1}{6}(x-2)^3$
B. $\frac{1}{12}(x-2)^3$
C. $\frac{1}{24}(x-2)^3$
D. $-\frac{1}{3}(x-2)^3$
E. $-(x-2)^3$

20. Using the fact that $\dfrac{1}{1-x} = \displaystyle\sum_{k=0}^{\infty} x^k$, $|x| < 1$, what is the coefficient of the term containing x^6 in the power series representation of $f(x) = \dfrac{1}{(1-x^2)^2}$?

A. 1
B. 2
C. 3
D. 4
E. 5

21. Using the fact that $\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1}$, $|x| \leq 1$, use the first two terms of the Maclaurin series for $3x^3 \tan^{-1}(2x)$ to estimate

$$\int_0^1 3x^3 \tan^{-1}(2x)\, dx$$

A. $\frac{37}{50}$
B. $\frac{1}{64}$
C. $\frac{84}{35}$
D. $\frac{2}{35}$
E. $\frac{3}{5}$

22. The center and radius of the circle $r = -6\sin\theta + 8\cos\theta$ in polar coordinates is

A. (4,3) and 5
B. (-6,8) and 10
C. (8,-6) and 10
D. (-3,4) and 5
E. (4,-3) and 5

23. The arc length of the cardioid $r = 3 + 3\cos\theta$ for $0 \leq \theta \leq 2\pi$.

A. 16

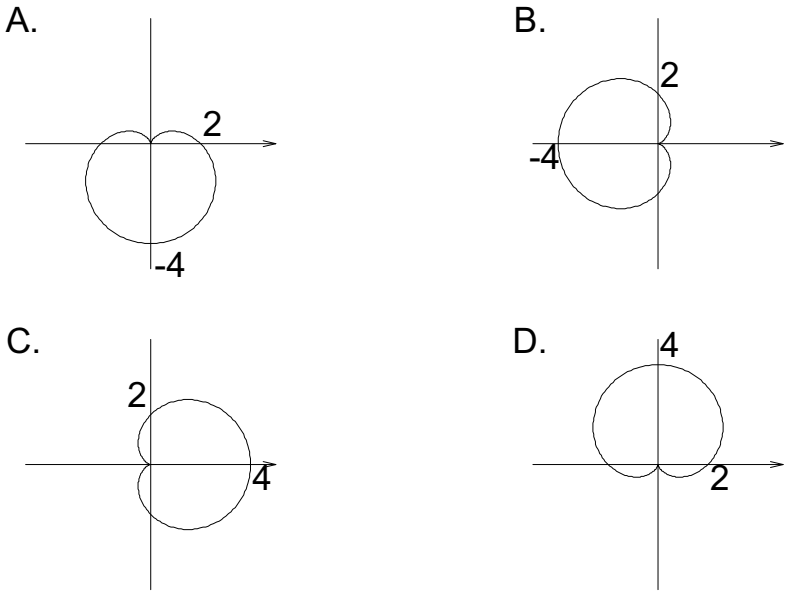
B. 8

C. 24

D. 12

E. 6

24. Which of the following pictures represents the curve given by the equation $r = 2 + 2\sin\theta$ with $-\pi \leq \theta \leq \pi$ in polar coordinates ?



A.

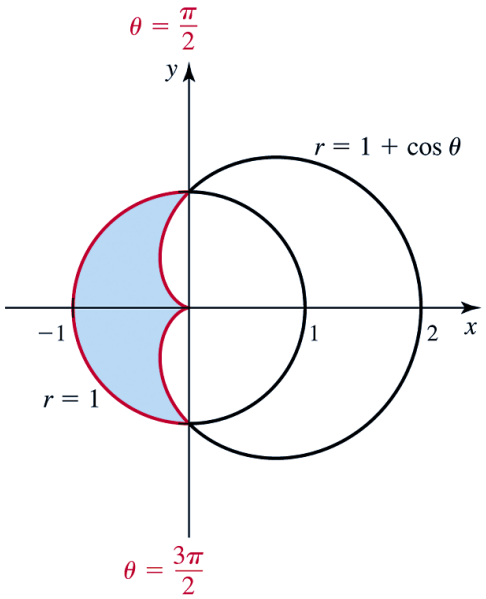
B.

C.

D.

E. None of the above

25. The area of the shaded region is



A. $\frac{1}{2}\int_{\pi/2}^{\pi}[1-(1+\cos\theta)^2]d\theta$

B. $\int_{\pi/2}^{\pi}(1+\cos\theta)^2 d\theta$

C. $\frac{\pi}{4}-\frac{1}{2}\int_{\pi/2}^{\pi}(1+\cos\theta)^2 d\theta$

D. $\frac{\pi}{2}-\int_{\pi/2}^{\pi}(1+\cos\theta)^2 d\theta$

E. $\frac{\pi}{2}-\int_{0}^{\pi}(1+\cos\theta)^2 d\theta$