MA 16200

FINAL EXAM

12/16/2021

TEST/QUIZ NUMBER: **1010**

NAME ____________________ YOUR TA'S NAME ____________________

STUDENT ID # ____________________ RECITATION TIME ____________________

You must use a #2 pencil on the scantron sheet. Write **1010** in the TEST/QUIZ NUMBER boxes and blacken in the appropriate digits below the boxes. On the scantron sheet, fill in your **TA**'s name for the INSTRUCTOR and **MA 162** for the COURSE number. Fill in whatever fits for your first and last NAME. The STUDENT IDENTIFICATION NUMBER has ten boxes, so use **00** in the first two boxes and your PUID in the remaining eight boxes. Fill in your three-digit SECTION NUMBER. If you do not know your section number, ask your TA. Complete the signature line.

There are **25** questions, each worth 4 points. This is a two-hour exam. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work in this exam booklet and indicate your answers in the booklet in case the scantron is lost. Use the back of the test pages for scrap paper. Turn in both the scantron sheet and the exam booklet when you are finished.

If you finish the exam before 5:20, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 3:50. If you don't finish before 5:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

1. $\displaystyle\int e^{3x} \sin x \, dx$

A. $\dfrac{e^{3x}(\sin x + 3\cos x)}{8} + C$

B. $\dfrac{e^{3x}(\sin x + \cos x)}{10} + C$

C. $\dfrac{e^{3x}(3\sin x - \cos x)}{10} + C$

D. $\dfrac{e^{3x}(\sin x + 3\cos x)}{10} + C$

E. $\dfrac{e^{3x}(\sin x + \cos x)}{8} + C$

F. $\dfrac{e^{3x}(\cos x - 3\sin x)}{8} + C$

2. $\displaystyle\int_1^4 \frac{\sqrt{x^2 + 4x - 5}}{x + 2} \, dx$

A. $3\sqrt{3} - \pi$

B. $\sqrt{3}$

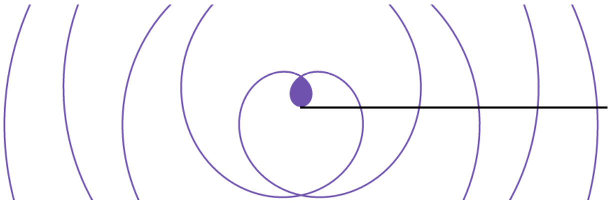
C. $\dfrac{1}{2}$

D. $\dfrac{\sqrt{3}}{2}$

E. $\sqrt{3} - \dfrac{\pi}{3}$

F. $\dfrac{\sqrt{3}}{3}$

3. The graph below is $r = \theta$. Find the area of the small shaded region.



A. $\dfrac{4\pi^3}{3}$

B. $\dfrac{\pi^3}{3}$

C. $\dfrac{\pi^3}{48}$

D. $\dfrac{\pi^3}{24}$

E. $\dfrac{\pi^3}{12}$

F. $\dfrac{\pi^3}{6}$

4. Find the function $f(x)$ represented by the following power series

$$\sum_{k=0}^{\infty} \frac{x^{2k+3}}{2^k}$$

A. $f(x) = \dfrac{x^3}{2 - x^2}$ for $|x| < \sqrt{2}$.

B. $f(x) = \dfrac{2}{2 - x^2}$ for $|x| < \sqrt{2}$.

C. $f(x) = \dfrac{2x^3}{2 - x^2}$ for $|x| < \sqrt{2}$.

D. $f(x) = \dfrac{2}{2 - x}$ for $|x| < 2$.

E. $f(x) = \dfrac{2x^3}{2 - x}$ for $|x| < 2$.

F. $f(x) = \dfrac{x^3}{2 - x}$ for $|x| < 2$.

5. Find the volume of the solid generated when the region bounded by the following two curves is revolved about the y-axis: $y = x - x^2, \quad y = 0$.

A. $\dfrac{11\pi}{30}$

B. $\dfrac{2\pi}{15}$

C. $\dfrac{\pi}{3}$

D. $\dfrac{5\pi}{6}$

E. $\dfrac{\pi}{30}$

F. $\dfrac{\pi}{6}$

6. Find the length of the "cardioid" curve given by

$$r = 1 + \cos\theta$$

A. 4π

B. 8

C. 4

D. 2π

E. $5\pi/4$

F. 6

7. Convert the polar equation

$$r = \frac{1}{\cos\theta + \sin\theta}$$

to a Cartesian equation.

A. $x^3 + x^2y + xy^2 + y^3 = 1$

B. $\sqrt{x^2+y^2} = \dfrac{1}{x+y}$

C. $y = 1 - x$

D. $y = \sqrt{1-x^2}$

E. $\left(x - \dfrac{1}{2}\right)^2 + \left(y - \dfrac{1}{2}\right)^2 = \dfrac{1}{2}$

F. $y = x + 1$

8. Which of the following represents $(x, y) = (-\sqrt{3}, -3)$ in polar coordinates?

A. $(r,\theta) = \left(-\sqrt{3}, \dfrac{\pi}{3}\right)$

B. $(r,\theta) = \left(-2\sqrt{3}, -\dfrac{\pi}{3}\right)$

C. $(r,\theta) = \left(\sqrt{3}, -\dfrac{\pi}{3}\right)$

D. $(r,\theta) = \left(2\sqrt{3}, \dfrac{\pi}{3}\right)$

E. $(r,\theta) = \left(2\sqrt{3}, \dfrac{4\pi}{3}\right)$

F. $(r,\theta) = \left(-2\sqrt{3}, \dfrac{4\pi}{3}\right)$

9. $\displaystyle\int_0^{\pi/4} \sin^2\theta\cos^2\theta\, d\theta$

A. $\dfrac{3\pi}{32}$

B. $\dfrac{\pi}{32}$

C. $\dfrac{3\pi+8}{32}$

D. $\dfrac{\pi+1}{32}$

E. $\dfrac{\pi-1}{32}$

F. $\dfrac{3\pi-8}{32}$

10. Evaluate (find the sum) of the following series:

$$\sum_{k=0}^{\infty} \frac{1}{2^{2k+1}}$$

A. $\dfrac{3}{4}$

B. $\dfrac{2}{3}$

C. $\dfrac{3}{2}$

D. $\dfrac{4}{3}$

E. $\dfrac{1}{2}$

F. 1

11. Find a series expression for
$$\int_0^1 \tan^{-1}(x^2)\, dx$$

Hint: $\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$ for $|x| \leq 1$.

A. $\frac{1}{2 \cdot 1} - \frac{1}{4 \cdot 3} + \frac{1}{6 \cdot 5} - \frac{1}{8 \cdot 7} + \frac{1}{10 \cdot 9} - \dots$

B. $\frac{1}{3 \cdot 2} - \frac{1}{7 \cdot 4} + \frac{1}{11 \cdot 6} - \frac{1}{15 \cdot 8} + \frac{1}{19 \cdot 10} - \dots$

C. $\frac{1}{3 \cdot 2} - \frac{1}{5 \cdot 4} + \frac{1}{7 \cdot 6} - \frac{1}{9 \cdot 8} + \frac{1}{11 \cdot 10} - \dots$

D. $\frac{1}{3 \cdot 1} - \frac{1}{5 \cdot 3} + \frac{1}{7 \cdot 5} - \frac{1}{9 \cdot 7} + \frac{1}{11 \cdot 9} - \dots$

E. $\frac{1}{2 \cdot 1} - \frac{1}{6 \cdot 3} + \frac{1}{10 \cdot 5} - \frac{1}{14 \cdot 7} + \frac{1}{18 \cdot 9} - \dots$

F. $\frac{1}{3 \cdot 1} - \frac{1}{7 \cdot 3} + \frac{1}{11 \cdot 5} - \frac{1}{15 \cdot 7} + \frac{1}{19 \cdot 9} - \dots$

12. Five of these series converge, and one diverges. Which series **diverges**?

A. $\frac{1}{4^2} + \frac{2}{5^2} + \frac{3}{6^2} + \frac{4}{7^2} + \frac{5}{8^2} + \frac{6}{9^2} + \dots$

B. $\frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \frac{2^5}{5!} + \frac{2^6}{6!} + \dots$

C. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \frac{1}{6\sqrt{6}} + \dots$

D. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots$

E. $0.9 + (0.9)^2 + (0.9)^3 + (0.9)^4 + (0.9)^5 + (0.9)^6 + \dots$

F. $-\frac{1}{\ln 1.2} + \frac{1}{\ln 1.4} - \frac{1}{\ln 1.6} + \frac{1}{\ln 1.8} - \frac{1}{\ln 2} + \frac{1}{\ln 2.2} - \dots$

13. A cylindrical tank (resting on its circular base) has height 4m and radius 1m. If the tank is **half** full of water, how much work is required to pump all the water out the top? Use ρ for the liquid density ($\mathrm{kg/m^3}$) and g for the gravitation constant (N/kg).

A. $10\pi\rho g$ N·m

B. $12\pi\rho g$ N·m

C. $4\pi\rho g$ N·m

D. $2\pi\rho g$ N·m

E. $8\pi\rho g$ N·m

F. $6\pi\rho g$ N·m

14. $\vec{\boldsymbol{a}} = \langle \frac{5}{2}, 3, 4\rangle$ and $\vec{\boldsymbol{b}} = \langle 1, 2, 2\rangle$. Find a unit vector normal to $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$. ("Normal" means orthogonal to $\vec{\boldsymbol{a}}$ and orthogonal to $\vec{\boldsymbol{b}}$)

A. $\left\langle -\frac{2}{5}, -\frac{1}{5}, \frac{2}{5} \right\rangle$

B. $\left\langle -\frac{4}{9}, -\frac{2}{9}, \frac{4}{9} \right\rangle$

C. $\left\langle -\frac{4}{33}, -\frac{2}{33}, \frac{4}{33} \right\rangle$

D. $\left\langle \frac{4}{9}, \frac{2}{9}, -\frac{4}{9} \right\rangle$

E. $\left\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$

F. $\left\langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right\rangle$

15. Find the length of the curve $y = \ln(\sec x)$ from $(0,0)$ to $\left(\frac{\pi}{4}, \frac{\ln 2}{2}\right)$

A. $\ln(1+\sqrt{2})$

B. $\sqrt{4-2\sqrt{2}}$

C. $\sqrt{2}$

D. $\frac{\pi}{6} + \frac{1}{2}\ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

E. $\frac{\pi + \ln 4}{4}$

F. $\frac{\pi + 4}{4}$

16. Recall that $\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$ for all x, and $\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$ for $|x| \le 1$.

Find the limit

$$\lim_{x\to 0} \frac{\cos(x^3) + \tan^{-1}\left(\frac{x^6}{2}\right) - 1}{x^{12}}$$

A. $\frac{1}{4}$

B. $-\frac{7}{24}$

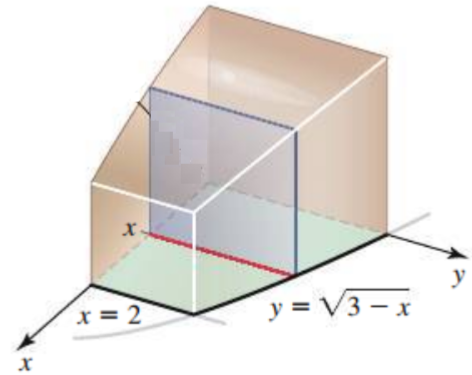
C. ∞

D. $\frac{1}{24}$

E. $-\frac{1}{3}$

F. 0

17. Find the volume of a solid whose base is the region in the first quadrant bounded by the curve $y = \sqrt{3-x}$ and the line $x = 2$, and whose cross sections through the solid perpendicular to the x-axis are squares.



A. $2\sqrt{3}$

B. 6

C. $2\sqrt{3} - \dfrac{2}{3}$

D. 2

E. $\dfrac{2}{3}$

F. 4

18. Find the Taylor Series for $f(x) = \sin(\pi x)$ centered at $a = 1$.

A. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k+1}\pi^k}{k!}(x-1)^k$

B. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k+1}\pi^{2k+1}}{(2k+1)!}(x-1)^{2k+1}$

C. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k}\pi^k}{(2k+1)!}(x-1)^{2k+1}$

D. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k}\pi^{2k+1}}{(2k+1)!}(x-1)^{2k+1}$

E. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k}\pi^k}{k!}(x-1)^k$

F. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^{k+1}\pi^k}{(2k+1)!}(x-1)^{2k+1}$

19. Describe the following two series: $\displaystyle\sum_{k=1}^{\infty}(-1)^k \tan\left(\frac{1}{k}\right)$ and $\displaystyle\sum_{k=2}^{\infty}\left(\frac{-1}{\ln k}\right)^k$

A. One series converges conditionally, and one series converges absolutely.

B. Both series converge conditionally.

C. One series diverges, and one series converges conditionally.

D. Both series converge absolutely.

E. One series diverges, and one series converges absolutely.

F. Both series diverge.

20. The angle between vector $\vec{\boldsymbol{v}}$ and vector $\vec{\boldsymbol{w}}$ is $\dfrac{\pi}{3}$. Suppose $\vec{\boldsymbol{w}} \cdot \vec{\boldsymbol{u}} = 6$ where $\vec{\boldsymbol{u}}$ is the unit vector that points in the direction of $\vec{\boldsymbol{v}}$. Find $|\vec{\boldsymbol{w}}|$.

A. 3

B. 6

C. $6\sqrt{3}$

D. 12

E. $6\sqrt{2}$

F. $4\sqrt{3}$

21. $\displaystyle\int \frac{3x^2+2x+3}{x^4+2x^2+1}\,dx$

A. $\tan^{-1}\left(\dfrac{1}{x^2+1}\right)+C$

B. $\dfrac{3\tan^{-1}x}{x^2+1}+C$

C. $\dfrac{\tan^{-1}x}{x^2+1}+C$

D. $3\tan^{-1}\left(\dfrac{1}{x^2+1}\right)+C$

E. $\tan^{-1}x-\dfrac{3}{x^2+1}+C$

F. $3\tan^{-1}x-\dfrac{1}{x^2+1}+C$

22. Suppose $p_2(x)$ is the Taylor polynomial of order $n=2$ centered at $a=1$ for the function

$$f(x)=x^{10}.$$

Compute $p_2(0)$.

A. 0

B. 1

C. 101

D. 81

E. 56

F. 36

23. Find the interval of convergence for the following power series:

$$\sum_{k=1}^{\infty} \frac{x^k}{\sqrt{k}}$$

A. Only $x = 0$

B. $(-\infty, \infty)$

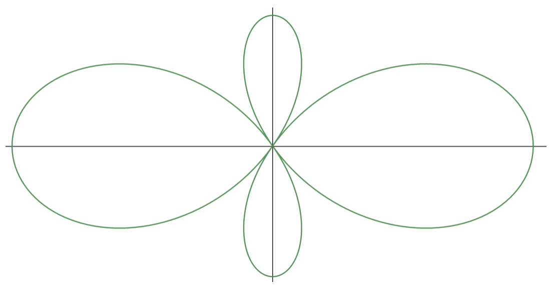
C. $[-1, 1)$

D. $(-1, 1)$

E. $[-1, 1]$

F. $(-1, 1]$

24. Find the polar equation with a graph given by the following curve:



A. $r = 1 + 3\sin(2\theta)$

B. $r = 1 + 2\sin(3\theta)$

C. $r = 2 + 3\cos(\theta)$

D. $r = 1 + 2\cos(3\theta)$

E. $r = 2 + 3\sin(\theta)$

F. $r = 1 + 3\cos(2\theta)$

25. The MacLaurin series for $\tan x$ is as follows:

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \ldots, \text{ for } |x| < \frac{\pi}{2}$$

Which of the following is the MacLaurin series for $\sec^2(x^3)$?

A. $1 + x^5 + \dfrac{2x^7}{3} + \dfrac{17x^9}{45} + \ldots$

B. $1 + x^6 + \dfrac{2x^{12}}{3} + \dfrac{17x^{18}}{45} + \ldots$

C. $3x^2 + 3x^8 + 2x^{14} + \dfrac{17x^{20}}{15} + \ldots$

D. $1 + \dfrac{x^6}{2} + \dfrac{x^{12}}{12} + \dfrac{x^{18}}{45} + \ldots$

E. $x^3 + x^5 + \dfrac{2x^7}{3} + \dfrac{119x^9}{315} + \ldots$

F. $\dfrac{x^4}{6} + \dfrac{x^{10}}{36} + \dfrac{x^{16}}{135} + \dfrac{17x^{22}}{7560} + \ldots$