MA 16200
FINAL EXAM Form 01
December 11, 2019

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes): **01**

You must use a #2 pencil on the mark–sense sheet (answer sheet). On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.

There are **25** questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 7:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

Potentially Useless Formulas

$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \sec^3\theta\, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

1. What is the angle, in radians, between the vectors $< 3, 0, -3 >$ and $< 1, -1, 0 >$?

A. 0

B. $\dfrac{3\pi}{4}$

C. $\dfrac{5\pi}{6}$

D. $\dfrac{\pi}{6}$

E. $\dfrac{\pi}{3}$

2. Find the area of the parallelogram with vertices $A(0, 0, 0)$, $B(1, 1, 1)$, $C(1, 2, 1)$, and $D(2, 3, 2)$

A. 1

B. $\sqrt{2}$

C. $\sqrt{3}$

D. $\sqrt{5}$

E. $\sqrt{6}$

3. The base of a solid S is a triangular region with vertices at $(0,0)$, $(0,1)$, and $(2,1)$. If the cross-sections perpendicular to the x-axis are squares, calculate the volume of S.

A. $\frac{5}{6}$

B. $\frac{5}{8}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

E. $\frac{3}{4}$

4. Find the area of the region bounded by $y = \cos 2x$, $y = \sin 4x$, between $x = 0$ and $x = \frac{\pi}{4}$. You may need to use the double-angle identity $\sin 2\theta = 2 \sin \theta \cos \theta$

A. 1

B. 2

C. $\frac{1}{8}$

D. $\frac{1}{4}$

E. $\frac{1}{6}$

5. The area between the graphs of $y = x^2$ and $y = 2x$ is revolved around the x-axis. If the method of cylindrical shells is used, the integral representing the volume of the resulting solid is

A. $\displaystyle\int_0^4 2\pi y(y - \frac{y^2}{4})\ dy$

B. $\displaystyle\int_0^4 \pi(y^4 - 4y^2)\ dy$

C. $\displaystyle\int_0^2 2\pi y(y^2 - 2y)\ dy$

D. $\displaystyle\int_0^4 2\pi y(\sqrt{y} - \frac{y}{2})\ dy$

E. $\displaystyle\int_0^2 \pi(y^2 - 2y)\ dy$

6. What value of the positive constant c makes the volume of the solid obtained by rotating the area between $x = 0$, $y = e^{3x}$, and $x = c$ about the x-axis equal to π?

A. $\frac{1}{6}\ln 2$

B. $\frac{1}{3}\ln \pi$

C. $\frac{1}{6}\ln 7$

D. $\frac{1}{7}\ln 6$

E. $\frac{1}{6}\ln \pi$

7. Find the integral: $\int_1^e x^{1/2} \ln x \, dx$

A. $\frac{1}{9}e^{3/2} + \frac{4}{9}$

B. $\frac{2}{9}e^{3/2} + \frac{4}{9}$

C. $\frac{4}{9}e^{3/2} + \frac{4}{9}$

D. $\frac{5}{9}e^{3/2} + \frac{4}{9}$

E. $\frac{7}{9}e^{3/2} + \frac{4}{9}$

8. Which of the following is the partial fraction expansion of $\frac{4x^3 - x}{x^2(x^2+1)^2}$?

A. $\frac{A}{x^2} + \frac{B}{x^2+1} + \frac{C}{(x^2+1)^2}$

B. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2+1} + \frac{D}{(x^2+1)^2}$

C. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$

D. $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} + \frac{E}{(x^2+1)^2}$

E. $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$

9. Use a long division to compute $\displaystyle\int \frac{x^2}{x+2}\, dx$

A. $\dfrac{x^2}{2} - 2x + 4\ln|x+2| + C$

B. $\dfrac{1}{6}x^3 + 2\ln|x+2| + C$

C. $\dfrac{x^2}{2} + \dfrac{1}{6}x^3 + C$

D. $x + 4\ln|x+2| + C$

E. $\dfrac{1}{3}x^3 + \ln|x+2| + C$

10. Which of the following statements are true?

I. $u = \tan x$ is an appropriate substitution for the integral $\displaystyle\int \tan^3 x \sec^4 x\, dx$

II. $u = \sin(3x)$ is an appropriate substitution for the integral $\displaystyle\int \cos^5(3x)\sin^4(3x)\, dx$

III. $\displaystyle\int_0^{\pi/2} \sin^2\theta\, d\theta = \frac{\pi}{4}$

A. I and II only

B. I and III only

C. II and III only

D. None is true

E. All are true

11. What does the integral $\int \frac{x^2}{\sqrt{x^2+25}}\,dx$ become after a trigonometric substitution?

A. $25\int(\tan^2\theta)(\sec\theta)\,d\theta$

B. $5\int(\tan^2\theta)(\sec\theta)\,d\theta$

C. $25\int\frac{\tan^2\theta}{\sec\theta}\,d\theta$

D. $5\int\frac{\tan^2\theta}{\sec\theta}\,d\theta$

E. $25\int\sin^2\theta\,d\theta$

12. A spring has a natural length of 10 m. If the **work** done in stretching the spring from 10 m (its natural length) to 11 m is $\frac{5}{2}$ joules, what is the **force** in newtons needed to stretch the spring from its natural length to a length of 20 m?

A. 100

B. 125

C. 50

D. 25

E. 5

13. How many of the following statements are FALSE?

I. If f is continuous and $0 < f(x) < g(x)$ on the interval $[0, \infty)$, and $\displaystyle\int_0^\infty g(x)\,dx = M < \infty$, then $\displaystyle\int_0^\infty f(x)\,dx$ exists.

II. If $\displaystyle\lim_{x\to\infty} f(x) = 1$, then $\displaystyle\int_0^\infty f(x)\,dx$ exists.

III. If $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$ exists, then $\displaystyle\int_0^1 \frac{1}{x^q}\,dx$ exists, where $q > p$.

IV. If $\displaystyle\int_0^\infty \frac{1}{x^p}\,dx$ exists, then $\displaystyle\int_0^\infty \frac{1}{x^q}\,dx$ exists, where $q > p$.

A. 0

B. 1

C. 2

D. 3

E. 4

14.

$$I.\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}-3}{n};\quad II.\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{[\ln(n+1)]^2};\quad III.\ \sum_{n=1}^{\infty}(-1)^{n-1}\sin(n).$$

Which of the following statements is true?

A. All three are convergent

B. Only II is convergent

C. I and II are convergent

D. II and III are convergent

E. All three are divergent

15. Determine if the geometric series converges or diverges. If it converges, find its sum.

$$\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \cdots$$

A. Converges to $\frac{2}{3}$

B. Converges to 2

C. Converges to $\frac{1}{2}$

D. Converges to $\frac{3}{2}$

E. Diverges

16. Let S_N be the N-th partial sum of the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{2n-1}$$

Compute $S_{50} - S_{49}$

A. $-\dfrac{1}{50}$

B. $-\dfrac{1}{99}$

C. $\dfrac{1}{97}$

D. $\dfrac{1}{101}$

E. 1

17. Which of the following statements are correct?

I. $\displaystyle\sum_{n=9001}^{\infty} \frac{\cos(n\pi)}{n^2}$ converges conditionally

II. The ratio test can be used to show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{10}}$ converges

III. A series is convergent if the sequence of its partial sums is bounded

A. All of them

B. None of them

C. I and II only

D. II and III only

E. III only

18. Find the interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-4)^n}{n^5 3^n}$

A. $[3, 5]$

B. $(-7, 7)$

C. $(-\infty, 7)$

D. $[1, 7]$

E. $(1, 7)$

19. Find the Taylor series for $f = \dfrac{1}{x}$ centered at $x = 3$

A. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{3^{n+1}}$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{3^{n}}$

C. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(x-3)^n}{3^{n+1}}$

D. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(x-3)^n}{3^{n}}$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+3)^n}{3^{n+1}}$

20. Find the Taylor polynomial for $f(x) = \dfrac{1}{2-x}$ of degree 3 centered at $x = 0$. Recall that if $|x| < 1$ then $\displaystyle\sum_{k=0}^{\infty} x = \frac{1}{1-x}$.

A. $\frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \frac{x^4}{16}$

B. $\frac{x}{2} - \frac{x^2}{4} + \frac{x^3}{8} - \frac{x^4}{16}$

C. $\frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} - \frac{x^3}{16}$

D. $\frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16}$

E. $\frac{1}{2} + \frac{x}{4} + \frac{x^2}{16} + \frac{x^3}{96}$

21. Which of the following is equal to $\int_0^1 \frac{\sin x}{x}\,dx$? Recall that $\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for all x.

A. $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(4n+1)(2n+1)!}$

B. $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+1)(2n+1)!}$

C. $\sum_{n=0}^{\infty}(-1)^n \frac{2}{(8n+1)(2n+1)!}$

D. $\sum_{n=0}^{\infty}(-1)^n \frac{2}{(2n+1)(2n+1)!}$

E. $\sum_{n=0}^{\infty}(-1)^n \frac{2}{(4n+1)(2n+1)!}$

22. In polar coordinates, the equation $r = \sin(\theta) + 2\cos(\theta)$ represents a circle with radius

A. $\frac{1}{2}$

B. $\frac{\sqrt{2}}{2}$

C. $\frac{\sqrt{3}}{2}$

D. 1

E. $\frac{\sqrt{5}}{2}$

23. Find the length of the following curve

$$r = e^{4\theta}, \qquad 0 \leq \theta \leq \pi$$

A. $\frac{1}{16}(e^{8\pi} - 1)$

B. $\frac{\sqrt{17}}{4}(e^{4\pi} - 1)$

C. $\frac{32}{3}(\pi^2 - 1)$

D. $e^{8\pi} - \sqrt{3}$

E. $\sqrt{17}(e^{4\pi} - 1)$

24. Find the area of the region inside both $r = 8\cos(\theta)$ and $r = 8\sin(\theta)$. You may find the following results useful in your calculation: $\displaystyle\int \cos^2(x)\ dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$, $\displaystyle\int \sin^2(x)\ dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

A. $8(\pi - 2)$

B. $8(\pi + 2)$

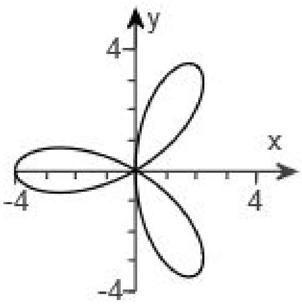
C. $\dfrac{8}{3}(4\pi - 3\sqrt{3})$

D. $4(2 - \sqrt{2})$

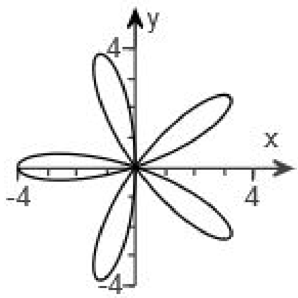
E. 16π

25. Which of the following is the graph of $r = -4\cos(3\theta)$?

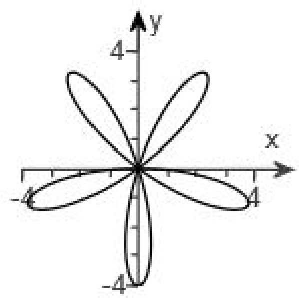
A



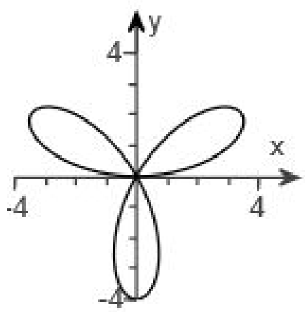
B



C



D



E