MA 16200
FINAL EXAM INSTRUCTIONS
VERSION 01
December 12, 2018

Your name ____________________ Your TA's name ____________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Write down YOUR NAME and TA's NAME on the exam booklet.

8. There are 25 questions, each worth 8 points. The total is $8 \times 25 = 200$. Blacken your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the scantron sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

11. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

12. If you finish the exam before 8:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 8:55, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Questions

1. The area of the region bounded by the curves $x = 2y$ and $x = y^2$ is

A. 2/3

B. 8/3

C. 4/3

D. 5/6

E. 11/6

2. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = -1$.

A. $\pi/2$

B. $\pi/3$

C. 2π

D. $-\pi/2$

E. π

3. A force of 4 Newtons stretches a spring with natural length of 10 cm to 20 cm. Find the total work by stretching the spring from its nature length 10 cm to 60 cm.
Hint: $J = N \cdot m$.

A. 6 J

B. 5 J

C. 3 J

D. 8 J

E. 3/2 J

4. Compute the value of the following definite integral: $\displaystyle\int_{-\pi/2}^{\pi/2} x \cos 2x \, dx$

A. 1/2

B. $-1/2$

C. -1

D. 0

E. 1

5. Evaluate the integral $\displaystyle\int_0^3 \sqrt{9-x^2}\,dx$

A. $3\pi/2$

B. $9\pi/2$

C. π

D. $3\pi/2 - 1$

E. $9\pi/4$

6. The arc of the parabola $y = x^2$ with $0 \le x \le 1$ is rotated about the y-axis. Find the area of the resulting surface.

A. $\frac{\pi}{6}(5^{\frac{3}{2}} - 1)$

B. $\pi(5\sqrt{5} - 1)$

C. $\pi(\sqrt{5} - 1)$

D. $\frac{\pi}{6}(\sqrt{5} - 1)$

E. $\frac{\pi}{2}(\sqrt{5} - 1)$

7. Find the volume of the parallelepiped formed by

$$\boldsymbol{a} = (3, 1, 3), \quad \boldsymbol{b} = (1, 2, 1) \text{ and } \quad \boldsymbol{c} = (1, 3, -2).$$

A. 8
B. 16
C. 15
D. -15
E. 27

8. If $\boldsymbol{a} \cdot \boldsymbol{b} = 3$ and $\boldsymbol{a} \times \boldsymbol{b} = (1, 2, 2)$, find the angle (in radians) between $\boldsymbol{a}$ and $\boldsymbol{b}$.

A. $\pi/3$
B. $\pi/4$
C. $\pi/6$
D. π
E. $\pi/2$

9. Find the centroid of the region bounded by the curves $y = x$ and $y = x^2$.

A. $(\frac{1}{2}, -\frac{2}{5})$
B. $(\frac{1}{12}, \frac{1}{15})$
C. $(\frac{1}{6}, \frac{1}{2})$
D. $(\frac{1}{2}, \frac{2}{5})$
E. $(\frac{8}{5}, \frac{1}{2})$

10. The three roots of $z^3 + 1 = 0$ are

A. $1,\ \frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i},\ \frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}$
B. $-1,\ \frac{\sqrt{3}}{2} + \frac{1}{2}\mathrm{i},\ \frac{\sqrt{3}}{2} - \frac{1}{2}\mathrm{i}$
C. $-1,\ \frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i},\ \frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}$
D. $1,\ \frac{\sqrt{3}}{2} + \frac{1}{2}\mathrm{i},\ \frac{\sqrt{3}}{2} - \frac{1}{2}\mathrm{i}$
E. $-1,\ \frac{1}{2} + \frac{1}{2}\mathrm{i},\ \frac{1}{2} - \frac{1}{2}\mathrm{i}$

11. Compute the following limit: $\lim_{n\to\infty}\left(n\sin(\pi/n)+\frac{n!}{n^n}\right)$.

A. π

B. ∞

C. 0

D. The limit does not exist.

E. $\pi+1$

12. Find the coefficient of x^5 in the Maclaurin series for the function $f(x)=\dfrac{x^2+1}{x-2}$.

A. $-\frac{3}{64}$

B. $\frac{5}{64}$

C. $-\frac{1}{64}$

D. $\frac{3}{64}$

E. - $\frac{5}{64}$

13. The area under cardioid $r = 1 - \sin\theta,\ 0 \le \theta \le 2\pi$, is

A. $\pi/4$
B. $3\pi/2$
C. $\pi/3$
D. $\pi/2$
E. π

14. The conic $3x^2 + 2y^2 = 2$ has its foci at

A. $(\pm 1/\sqrt{3}, 0)$
B. $(0, \pm\sqrt{3})$
C. $(\pm\sqrt{3}, 0)$
D. $(0, \pm 1/\sqrt{3})$
E. $(0, \pm 1/3)$

15. Using Maclaurin series and the Estimation Theorem for alternating series, we can obtain the approximation

$$\int_0^{0.1} \frac{dx}{1+x^2} \approx 0.1 - \frac{(0.1)^3}{3} \text{ with error } \leq c. \text{ The value of } c \text{ is}$$

A. $(0.1)^7$

B. $\frac{(0.1)^5}{5}$

C. $\frac{(0.1)^7}{7}$

D. $(0.1)^5$

E. $(0.1)^6$

16. $\displaystyle\int_3^4 \frac{1}{x^2-3x+2}\, dx =$

A. $\ln \frac{3}{5}$

B. $\ln \frac{4}{3}$

C. $\ln \frac{5}{3}$

D. $\ln \frac{3}{4}$

E. $\ln \frac{8}{9}$

17. Find the arc length of the curve $y = \ln(1 - x^2)$ with $0 \leq x \leq \frac{1}{2}$.

A. $\ln 6 - \frac{1}{2}$
B. $\ln 3 - \frac{1}{2}$
C. $\frac{1}{2}$
D. $\ln 3 + \frac{1}{2}$
E. $\ln 3 - 2$

18. Find the interval of convergence for the Taylor series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^n}(x-5)^n$.

A. $(-\frac{1}{3}, \frac{1}{3})$
B. $(5 - \frac{e}{3}, 5 + \frac{e}{3})$
C. $(5 - \frac{e}{3}, 5 + \frac{e}{3}]$
D. $(5 - \frac{1}{3}, 5 + \frac{1}{3})$
E. $(-\infty, \infty)$

19. The coefficient of x^4 in Maclaurin series for $f(x) = 1/\sqrt{1-x^2}$ is

A. $-3/8$

B. $3/8$

C. 0

D. $-3/4$

E. $3/4$

20. Which of the following equations in polar coordinates represents the circle with center $(x_0, y_0) = (0, 3)$ and radius 3 ?

A. $r = 6\sin\theta$

B. $r = 3\cos\theta$

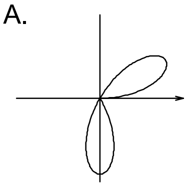
C. $r = 6\cos\theta$

D. $r = \cos 3\theta$

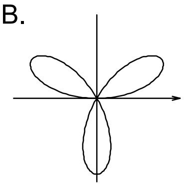
E. $r = 3\sin\theta$

21. Which of the following pictures represents the curve given by the equation $r = \sin 3\theta$ with $0 \le \theta \le \pi/3$ in polar coordinates ?

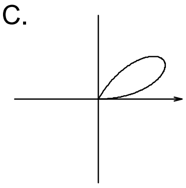
A.



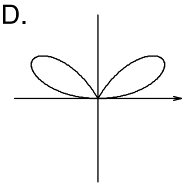
B.



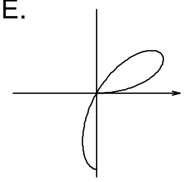
C.



D.



E.



22. The length of a parametric curve $\{x = \cos\theta + \theta\sin\theta,\ y = \sin\theta - \theta\cos\theta,\ 0 \le \theta \le \pi\}$ is

A. π

B. π^2

C. $2\pi/3$

D. $\pi/2$

E. $\pi^2/2$

23. Find the range of p (resp. q) such that the following series (a) (resp. (b)) converges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2p}+1}{\sqrt{n+2}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2q}}{\sqrt{n+2}}$

A. (a) no values for p; (b) no values for q

B. (a) $p < -\frac{1}{4}$; (b) $q < -\frac{1}{4}$

C. (a) all values for p; (b) all values for q

D. (a) no values for p; (b) $q < -\frac{1}{4}$

E. (a) $p < -\frac{1}{2}$; (b) $q < -\frac{1}{4}$

24. Which of the following series converge conditionally ?

I. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$

II. $\sum_{n=2}^{\infty}(-1)^n \frac{1}{\ln n}$

III. $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$

A. II and III only

B. I and II only

C. I and III only

D. All of them

E. None of them

25. The conic $4x^2 - 8x + y^2 + 4y + 4 = 0$ has its center at

A. $(1, 2)$
B. $(-1, -2)$
C. $(1, -2)$
D. $(-1, 2)$
E. $(2, 1)$

USEFUL FORMULAS

Trig Formulas:

$$\sin^2 x = \frac{1-\cos(2x)}{2}, \qquad \cos^2 x = \frac{1+\cos(2x)}{2}, \qquad \sec^2 x = 1 + \tan^2 x$$

Center of Mass:

$$\overline{x} = \frac{1}{A}\int_a^b x(f(x)-g(x))\,dx \qquad \text{and} \qquad \overline{y} = \frac{1}{A}\int_a^b \frac{1}{2}[(f(x))^2-(g(x))^2]\,dx$$

Arc Length, Surface Area and Volume:

Arc Length: $L = \int_a^b \sqrt{1+(f'(x))^2}\,dx$

Surface area: $S = 2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$ or $S = 2\pi\int_a^b x\sqrt{1+(f'(x))^2}\,dx$

Volume by the washer method: $V = \pi\int_a^b (R^2(x)-r^2(x))\,dx$; $R(x)$ and $r(x)$ are the longer and shorter radii of the washer. Volume by cylindrical shells: $V = 2\pi\int_a^b xf(x)\,dx$

Maclaurin Series:

The geometric series: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, provided $|x| < 1$

The exponential function: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x

Sine: $\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$ for all x. Cosine: $\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$ for all x

The binomial series: Provided $|x| < 1$,

$$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \ldots + \frac{k(k-1)(k-2)\ldots(k-n+1)}{n!}x^n \ldots$$

Conic sections:

Parabola: $x^2 = 4py$, focus at $(0,p)$, or $y^2 = 4px$, focus at $(p,0)$.

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $b^2 = a^2 - c^2$, foci at $(\pm c, 0)$, or $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, $b^2 = a^2 - c^2$, foci at $(0, \pm c)$

Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, foci at $(\pm c, 0)$, or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, foci at $(0, \pm c)$, where $c^2 = a^2 + b^2$

Asymptotes: $\frac{x}{a} = \pm\frac{y}{b}$ or $\frac{y}{a} = \pm\frac{x}{b}$

Shifted conic with center at (h,k): replace x by $x-h$ and y by $y-k$.