MA 16200 Final Exam, Test number 26, December 2017

Name ______________________________

10–digit PUID number ______________________________

Recitation Instructor ______________________________

Recitation Section Number and Time ______________________________

Instructions: **MARK TEST NUMBER 26 ON YOUR SCANTRON**

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 25 problems, each worth 8 points.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.
9. A collection of potentially useful identities:

$$\sin(a+b) = \sin a \cos b + \cos a \sin b \qquad 1 - \cos 2a = 2\sin^2 a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \qquad 1 + \cos 2a = 2\cos^2 a$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

1. A 100 pound cable, 20 feet in length, is hanging from the top of a tall building. Find the work needed to pull the cable to the top of the building.

   A. 100 ft-lb

   B. 1,000 ft-lb

   C. 500 ft-lb

   D. 2,000 ft-lb

   E. 800 ft-lb

2. The foci of the conic section given by $x^2 - 2x + 4y^2 = 3$ are at

   A. $(-2, 0), \quad (4, 0)$

   B. $(0, \sqrt{3}), \quad (0, 2 + \sqrt{3})$

   C. $(1 - \sqrt{3}, 0), \quad (1 + \sqrt{3}, 0)$

   D. $(0, 2), \quad (1, 3)$

   E. $(2, 0), \quad (4, 0)$

3. If $\boldsymbol{a} \cdot \boldsymbol{b} = \sqrt{3}$ and $\boldsymbol{a} \times \boldsymbol{b} = < 1, 2, 2 >$, find the angle (in radians) between $\boldsymbol{a}$ and $\boldsymbol{b}$.

A. π

B. $\pi/3$

C. $\pi/4$

D. $\pi/6$

E. $\pi/2$

4. In the Taylor series of e^{x^2} about 2, the coefficient of $(x-2)^2$ is

A. $9e^4$

B. $1/2$

C. $e^2/4$

D. $e^2/2$

E. $36e^4$

5. Find the volume generated when the region bounded by the curves $y = x$ and $x = y^2$ is rotated about the x-axis.

A. $5\pi/6$

B. $\pi/3$

C. $\pi/6$

D. $5\pi/3$

E. $2\pi/15$

6. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n} x^n$ is

A. $(-1, 1)$

B. $(-1, 1]$

C. $[-1, 1)$

D. $[-1, 1]$

E. None of the above.

7. A force of 4 Newtons stretches a spring with natural length of 0.1 m to 0.2 m. Find the work needed to stretch the spring from its natural length to 0.6 m.

A. 5 Joule

B. 3 Joule

C. 8 Joule

D. 6 Joule

E. 1.5 Joule

8. The length of the arc of the spiral given in polar coordinates by $r = e^{-2\theta}$, $0 \leq \theta \leq 1/2$, is

A. $(e+1)/\sqrt{6}$

B. $(1-e)/(2\sqrt{3})$

C. $4(1-e^{-1})$

D. $\sqrt{2}(e^2+1)$

E. $(e-1)\sqrt{5}/(2e)$

9. A conical tank is 5 meters high and the radius of its base is 2 meters long. The base of the tank rests on the ground. If the tank is filled with a liquid of density ρ kg/m^3 (and g is gravitational acceleration), the work necessary to empty it by pumping the liquid through its vertex at the top is

A. $20\pi\,\rho g$

B. $16\pi\,\rho g$

C. $50\pi\,\rho g$

D. $25\pi\,\rho g$

E. $12.5\pi\,\rho g$

10. Which is true? The series $\sum_{k=1}^{\infty} \frac{\ln k}{k^2}$ is

A. convergent by comparison with $\sum_{k=1}^{\infty} \frac{1}{k^2}$;

B. divergent by comparison with $\sum_{k=1}^{\infty} \frac{1}{k^2}$;

C. divergent by comparison with $\sum_{k=1}^{\infty} \frac{1}{k}$;

D. convergent by comparison with $\sum_{k=1}^{\infty} \frac{1}{k}$;

E. convergent by the integral test.

11. Given that $\int \sec x dx = \ln|\sec x + \tan x| + C$, the integral

$$\int \frac{1}{\sqrt{25+x^2}} dx$$

is equal to

A. $\ln|\sqrt{25+x^2} + \frac{x}{5}| + C$

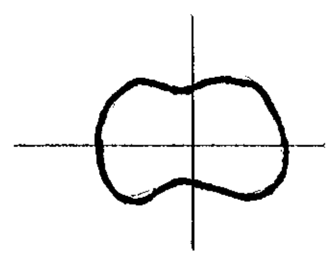
B. $\ln|\frac{1}{\sqrt{25+x^2}} + \frac{1}{x}| + C$

C. $\ln|\frac{x}{\sqrt{25+x^2}}| + C$

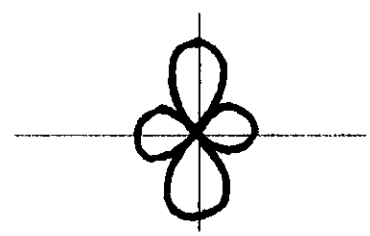
D. $\ln|\sqrt{25+x^2}| + C$

E. $\ln|\frac{\sqrt{25+x^2}}{5} + \frac{x}{5}| + C$

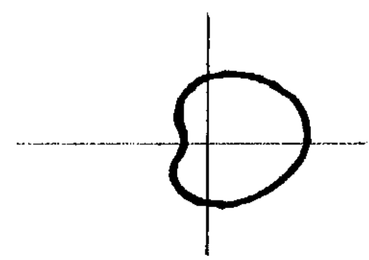
12. The polar equation $r = 3 + 2\cos 2\theta$ describes which of the following curves?



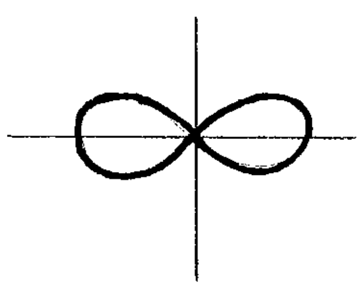
A.



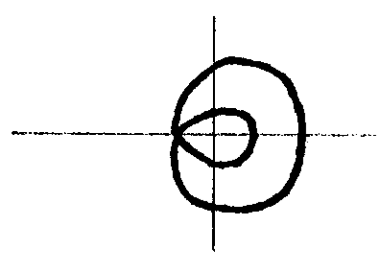
B.



C.



D.



E.

13. $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{x^2(x^2+1)} =$

A. $\pi/12$

B. $1 - 1/\sqrt{3} - \pi/12$

C. $1 - 1/\sqrt{3}$

D. $\sqrt{3} - 1$

E. $(\sqrt{3} - 1)/\pi$

14. $\displaystyle\frac{13}{3-2i} =$

A. $3 + 2i$

B. $-3 + (i/2)$

C. $(3 + i)/2$

D. $-2 + 3i$

E. $2 + 3i$

15. Find the proper form of partial fractions for the rational function

$$\frac{3x^4 - 2x^3 + 5}{(x^2 + 3x + 2)(x^2 - 4x + 4)(x^2 - 2x + 5)}.$$

You should factor the denominator as much as you can.

A. $\dfrac{Ax + B}{x^2 + 3x + 2} + \dfrac{Cx + D}{x^2 - 4x + 4} + \dfrac{Ex + F}{x^2 - 2x + 5}$

B. $\dfrac{A}{x + 2} + \dfrac{B}{x + 1} + \dfrac{C}{(x - 2)^2} + \dfrac{Dx + E}{x^2 - 2x + 5}$

C. $\dfrac{A}{x + 2} + \dfrac{B}{x + 1} + \dfrac{C}{(x - 2)^2} + \dfrac{D}{(x - 2)^3} + \dfrac{Ex + F}{x^2 - 2x + 5}$

D. $\dfrac{A}{x + 2} + \dfrac{B}{x + 1} + \dfrac{C}{(x - 2)^2} + \dfrac{Dx + E}{x^2 - 2x + 5}$

E. $\dfrac{A}{x + 2} + \dfrac{B}{x + 1} + \dfrac{C}{x - 2} + \dfrac{D}{(x - 2)^2} + \dfrac{Ex + F}{x^2 - 2x + 5}$

16. $\displaystyle\sum_{m=1}^{\infty} \frac{3 + 2^m}{2^{2m}} =$

A. 1

B. 4/3

C. 5/2

D. 3/2

E. 2

17. $\int_1^2 \frac{1}{x^2+3x+2}dx =$

A. $\ln\frac{8}{9}$

B. $\ln\frac{9}{8}$

C. $\ln\frac{1}{2}$

D. $\ln\frac{3}{4}$

E. $2\ln\frac{4}{3}$

18. A curve is given by parametric equations $x = t + \ln t$, $y = t - \ln t$. When $t = 1$ then $d^2y/dx^2 =$

A. 8/3

B. 7/4

C. 2/5

D. 1/4

E. 0

19. $\displaystyle\int_0^5 \frac{1}{x-2}dx =$

A. $\ln 3 - \ln 2$

B. $\ln 3 + \ln 2$

C. 0

D. $\ln 3$

E. The integral is divergent.

20. The radius of convergence of the power series $\displaystyle\sum_{j=0}^{\infty} \frac{2^j+1}{3^j+2}(x+2)^j$ is

A. $5/2$

B. $3/2$

C. 1

D. $2/3$

E. ∞

21. Find the arc length of the curve $y = \dfrac{x^2}{4} - \dfrac{\ln x}{2}$, $1 \le x \le 2$.

A. $\dfrac{3 + 2\ln 2}{4}$

B. $\ln 2 - \dfrac{3}{2}$

C. $\dfrac{1 + \ln 2}{2}$

D. $\dfrac{1}{2}$

E. $\ln 3 - 2$

22. The area between the spirals $r = \theta$ and $r = 2\theta$, $0 \le \theta \le \pi/2$, is

A. $\pi^2/4$

B. $\pi^2 - 2$

C. $\pi^3/16$

D. $\pi^3 + 4\pi$

E. $\pi^4/8$

23. Which is true? If a series

I. absolutely converges, then it converges; II. converges, then it converges absolutely; III. diverges, then it cannot converge conditionally.

A. Only I.

B. Only II.

C. Only I. and II.

D. Only II. and III.

E. Only I. and III.

24. Which is true? The point whose Cartesian coordinates are $(1, \sqrt{3})$ has polar coordinates

I. $(2, \pi/3)$ II. $(2, -5\pi/3)$ III. $(2, 4\pi/3)$

A. Only I.

B. Only II.

C. Only III.

D. Only I. and II.

E. All are true.

25. A homogeneous lamina of density 1 occupies the region bounded by the curves $y = x^2, \quad x = y^2$. Given that its moment about the y axis is $3/20$, its centroid is at

A. $(\frac{1}{3}, \frac{1}{3})$

B. $(\frac{1}{2}, \frac{9}{20})$

C. $(\frac{9}{20}, \frac{9}{20})$

D. $(\frac{1}{2}, \frac{1}{2})$

E. $(\frac{1}{20}, \frac{1}{4})$