

1. We want to approximate the sum

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 + 1}.$$

The alternating series error estimate guarantees that the error is $< 10^{-4}$, provided we add the first n terms, where $n =$

- A. 9
- B. 10
- C. 49
- D. 99
- E. None of the above is correct.

2. $\int_1^2 x \ln x \, dx =$

- A. $\ln 2 - 1$
- B. $\ln \sqrt{2} - 1/2$
- C. $2 \ln 2 - 3/2$
- D. $\ln 2 - 3/2$
- E. $2 \ln 2 - 3/4$

3. The length of the curve $x = e^t \cos 2t$, $y = e^t \sin 2t$, $0 \leq t \leq 1$, is

- A. 1
- B. 3
- C. $\sqrt{5} (e - 1)$
- D. $\sqrt{2} e$
- E. $6e - 2$

4. $\frac{2+i}{3-i} =$

- A. $2i$
- B. $2 - i$
- C. $1 + 2i$
- D. $1 - 2i$
- E. None of the above.

5. The vector of length 6 that points in the opposite direction to $2\vec{i} - \vec{j} + 2\vec{k}$ is

A. $-6\vec{i} + 3\vec{j} - 6\vec{k}$

B. $\vec{i} - \frac{\vec{j}}{2} + \vec{k}$

C. $-4\vec{i} + 2\vec{j} - 4\vec{k}$

D. $-2\sqrt{3}\vec{i} + \sqrt{3}\vec{j} - 2\sqrt{3}\vec{k}$

E. $-2\sqrt{6}\vec{i} + \sqrt{6}\vec{j} - 2\sqrt{6}\vec{k}$.

6. The Maclaurin series of $\sqrt{1 + 4x^2}$ is

A. $1 + 2x^2 - 2x^4 + 4x^6 + \dots$

B. $1 + 2x^2 - 3x^4 + 6x^6 + \dots$

C. $1 + 2x^2 - 2x^4 + 8x^6 + \dots$

D. $1 + 2x^2 - 3x^4 - 5x^6 + \dots$

E. $1 + 2x^2 - 2x^4 - 5x^6 + \dots$

7. $\int_1^{\infty} xe^{-x^2} dx =$

- A. $1/e^2$
- B. $1/(2e)$
- C. $1/e$
- D. $2/3$
- E. The integral diverges.

8. Evaluate $5 - \frac{25}{6} + \frac{125}{36} - \frac{625}{216} + \dots$

- A. $30/11$
- B. $6/11$
- C. $30/7$
- D. $15/11$
- E. $15/7$

9. Determine whether the following sequence and series converge:

$$\left\{ \frac{2n^2 - 1}{3n^2 + 8n} \right\}_{n=1}^{\infty}, \quad \sum_{n=1}^{\infty} \frac{2n^2 - 1}{3n^2 + 8n}.$$

- A. Both converge.
- B. Only the sequence converges.
- C. Only the series converges.
- D. Neither converge.
- E. None of the above is correct.

10. Consider the series

I. $1 - 1 + 1 - 1 + 1 - \dots,$

II. $\frac{\ln 3}{3} - \frac{\ln 6}{6} + \frac{\ln 9}{9} - \frac{\ln 12}{12} + \dots$

- A. By the alternating series test both are convergent.
- B. By the alternating series test both are divergent.
- C. The alternating series test is inconclusive for both.
- D. II. is convergent by the alternating series test, I. is divergent.
- E. I. is conditionally, II. is absolutely convergent.

11. If $\sum_{n=1}^{\infty} \frac{(x-1)^{3n}}{2+n+n^2}$ is the Taylor series of a function $g(x)$, then $g^{(6)}(1) =$

- A. 90
- B. 36
- C. -36
- D. -32
- E. 24

12. The area between the curves $y = 1 - 2x$ and $2 - 2x - x^2$ is

- A. $7/3$
- B. $4/3$
- C. $5/6$
- D. 2
- E. $3/2$

13. Which statement(s) is true?

I. The series $\sum_{k=2}^{\infty} \frac{1}{k \ln^q k}$ converges for $q = 1$.

II. The series $\sum_{k=2}^{\infty} \frac{1}{k \ln^q k}$ converges for $q = 2$.

- A. Only I;
- B. Only II;
- C. Both are true
- D. Neither is true
- E. None of the above is correct.

14. $\sum_{k=1}^{\infty} \frac{2^{k+4}}{k^2 2^k + k^3}$ is

- A. convergent by the ratio test;
- B. divergent by the ratio test;
- C. convergent by the comparison test;
- D. divergent by the comparison test;
- E. convergent by the root test.

15. Which is/are absolutely convergent?

I. $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$

II. $\sum_{j=1}^{\infty} \frac{(-1)^j}{j^2}$

III. $\sum_{k=1}^{\infty} \frac{(-1)^k e^k}{k^k}$

- A. All are.
B. None is.
C. Only I and II.
D. Only II and III.
E. Only II.

16. $\int \frac{2dx}{x^2(x-1)} =$

A. $\ln \left| \frac{x}{x-2} \right| - \frac{2}{x} + C$

B. $\ln \left| \frac{x-1}{x} \right| + \frac{1}{x} + C$

C. $2 \ln \left| \frac{x-1}{x} \right| + \frac{2}{x} + C$

D. $2 \ln \left| \frac{x}{x-1} \right| + \frac{1}{x} + C$

E. $\ln \left| \frac{x-1}{x} \right| - \frac{2}{x} + C$

17. Which integral arises when one uses a trigonometric substitution to compute

$$\int \frac{x^2 dx}{\sqrt{4+x^2}}?$$

- A. $\int 4 \tan^2 \theta \sec \theta d\theta$
- B. $\int 2 \tan^2 \theta \cos \theta d\theta$
- C. $\int 8 \tan^2 \theta \sec^2 \theta d\theta$
- D. $\int 4 \tan^2 \theta \cos \theta d\theta$
- E. $\int 4 \sec^3 \theta d\theta$

18. The radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-3)^{2n}}{2^n}$$

is

- A. $\sqrt{2}$
- B. $\sqrt{3}$
- C. 3
- D. 1
- E. ∞

19. The tangent line to the curve $x = \cos t + \sin t$, $y = e^{2t}$, corresponding to $t = 0$, has equation

A. $y = x$

B. $y = ex + 1 - e$

C. $y = x \ln 2 + 1 - \ln 2$

D. $y = \frac{3 - x}{2}$

E. $y = 2x - 1$

20. The polar coordinates of a point are $r = 2\sqrt{3}$, $\theta = \pi/3$. What are its Cartesian coordinates?

A. $(3, \sqrt{2})$

B. $(1, 3)$

C. $(\sqrt{3}, 3)$

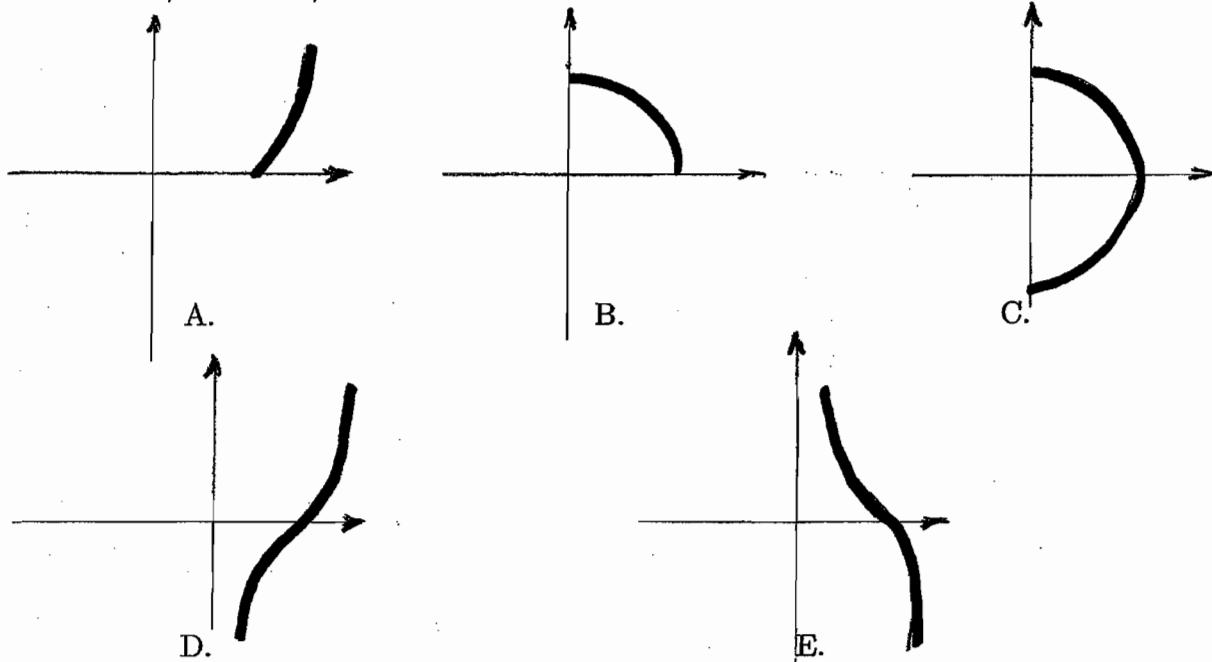
D. $(2\sqrt{3}, \sqrt{3}/2)$

E. None of the above.

21. $\lim_{n \rightarrow \infty} (n + 2)^{1/3n} =$

- A. $1/6$
- B. $1/3$
- C. 1
- D. 0
- E. The limit does not exist.

22. Which curve is represented by the parametric equations $x = \sec t$, $y = \tan^2 t$, $-\pi/2 < t < \pi/2$?



23. The cosine of the angle between the vectors $2\vec{i} - \vec{j} + \vec{k}$ and $\vec{i} + \vec{k}$ is
- A. $1/\sqrt{2}$
 - B. $\sqrt{3}/2$
 - C. $1/\sqrt{12}$
 - D. $3/\sqrt{6}$
 - E. $1/2$
24. Let D be the region in the xy plane bounded by $y = 2x - x^2$ and the x -axis. Find the volume of the solid obtained by revolving D about the y axis.
- A. $11\pi/6$
 - B. 2π
 - C. $8\pi/3$
 - D. $13\pi/4$
 - E. 3π

25. Which of the curves below corresponds to the polar equation $r = 2 + \cos 2\theta$?

