MA 162 FINAL EXAM **Fall 2002**

Name

Student ID Number

Lecturer

Recitation Instructor

Instructions:

1. This package contains 24 problems for a total of 200 points.
2. Please supply all information requested above and on the mark-sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes or calculator, please.
5. Some trigonometric formulas:

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

(8 pts) 1. Find the area of the triangle whose vertices are $(0,0,0)$, $(1,2,3)$, $(4,5,6)$.

A. 6

B. $\frac{3\sqrt{6}}{2}$

C. 3

D. $3\sqrt{6}$

E. 4

(8 pts) 2. Suppose the vectors $\vec{u} = \vec{i} - \vec{j} + \vec{k}$ and $\vec{v} = \vec{i} - a\vec{j} + a\vec{k}$ are perpendicular, then $a =$

A. 0

B. 1

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. -1

(8 pts) 3. Compute $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x dx$

A. $\frac{\pi}{16}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

E. π

(8 pts) 4. Compute $\int_0^{\frac{\pi}{4}} \tan x \sec^4 x dx$.

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. 1

E. $\frac{3}{5}$

(8 pts) 5. Compute $\int_0^1 \frac{x}{(x+1)^2}\, dx$.

A. $\ln 2$

B. $\ln 2 - \frac{1}{2}$

C. $\ln 2 + \frac{1}{2}$

D. $\ln\left(\frac{3}{2}\right)$

E. $\ln(3)$

(8 pts) 6. The substitution best suited for integrating $\int \frac{1}{\sqrt{4x^2+4x-3}}\,dx$ is

A. $x = \frac{1}{2}\sin u$

B. $x = \sin u - \frac{1}{2}$

C. $x = \sec u$

D. $x = \frac{1}{2}\sec u$

E. $x = \sec u - \frac{1}{2}$

(8 pts) 7. Compute the improper integral $\int_1^2 \frac{1}{(x-1)^{\frac{3}{2}}}\,dx$.

A. 2

B. -2

C. integral is divergent

D. 1

E. -1

(8 pts) 8. If it takes 4 ft-lbs of work to stretch a spring from neutral position to a distance 2 feet beyond, how much work is required to stretch the spring from 2 feet to 3 feet beyond neutral position?

A. 2 ft-lbs

B. 3 ft-lbs

C. 4 ft-lbs

D. 5 ft-lbs

E. 6 ft-lbs

(10 pts) 9. Let R be the region bounded by $y = \ln x$ $y = 0$, $x = 1$, $x = e$. What is the volume of the solid obtained by rotating R around the y axis?

A. $\frac{\pi}{2}(e^2 - 1)$

B. $\frac{\pi}{2}(e^2 + 1)$

C. $\frac{\pi}{2}(e^2 - 3)$

D. $\pi(e^2 + 1)$

E. $\pi(e^2 - 1)$

(10 pts) 10. Find the area of the surface of revolution obtained by rotating the curve $y = x^3$, $0 \leq x \leq 1$ about the x–axis.

A. $\frac{\pi}{27}(10^{\frac{3}{2}} - 1)$

B. $\frac{\pi}{54}(10^{\frac{3}{2}} - 1)$

C. $\frac{\pi}{9}(10^{\frac{3}{2}} - 1)$

D. $\frac{\pi}{27}$

E. $\frac{\pi}{9}$

(3 pts) 11. We wish to estimate $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$ to within 10^{-4}. Then the alternating series test says we must take $\sum_{n=2}^{k} \frac{(-1)^n}{(\ln n)^2}$ with k at least 100?

A. True

B. False

(10 pts) 12. If we write $\tan^{-1}(x) = \sum_{n=0}^{\infty} C_n(x-1)^n$, then $C_2 =$

A. 0

B. 1/2

C. 1/6

D. 1/4

E. −1/4

(7 pts) 13. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x-8)^n}{n4^n}$ is

A. $3 < x < 5$

B. $3 \leq x \leq 5$

C. $x = 4$ only

D. $2 < x < 6$

E. $2 \leq x < 6$

(8 pts) 14. $\lim_{n\to\infty} \frac{9^n(n+1)}{10^{n+1}(n+2)} =$

A. $\frac{9}{100}$

B. $\frac{9}{10}$

C. 0

D. $\frac{9}{20}$

E. the limit does not exist

(8 pts) 15. $\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n} =$

A. $\frac{3}{2}$

B. $\frac{9}{2}$

C. $\frac{15}{2}$

D. $\frac{5}{2}$

E. $\frac{7}{2}$

(10 pts) 16. Which of the following statements is true for the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$:

A. The series converges by the integral test

B. The series diverges by the integral test

C. The series converges since $\lim_{n\to\infty} \frac{1}{n(\ln n)^2} = 0$

D. The series converges by the ratio test

E. The series diverges by the ratio test

(10 pts) 17. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^p+3}}$ converges if and only if

A. $p > -1$

B. $p > 0$

C. $p > 1$

D. $p > 2$

E. $p > 3$

(10 pts) 18. Which of the following is the Maclaurin series of $\dfrac{2}{(1+x)^3}$?

A. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2} x^n$

B. $\displaystyle\sum_{n=0}^{\infty} (-1)^n (n+1)(n+2) x^n$

C. $\displaystyle\sum_{n=0}^{\infty} (-1)^{n-1} \frac{(n+1)(n+2)}{2} x^n$

D. $\displaystyle\sum_{n=0}^{\infty} (-1)^{n-1} (n+1)(n+2) x^n$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n$

(6 pts) 19. The series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n+5}{2n-5}\right)^n$ is

A. absolutely convergent

B. conditionally convergent

C. divergent

(8 pts) 20. Find the slope of the tangent line to $x = te^{-t},\ y = \dfrac{t^3}{3}$, at $t = 2$.

A. $\dfrac{1}{e^2}$

B. $4e^2$

C. $-4e^2$

D. $\dfrac{-4}{e^2}$

E. e^2

(10 pts) 21. The area inside the curve $r = 3\sin\theta$ and outside the curve $r = 1 + \sin\theta$ is given by

A. $\dfrac{1}{2}\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (8\sin^2\theta - 1 - 2\sin\theta)\, d\theta$

B. $\dfrac{1}{2}\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (4\sin^2\theta - 4\sin\theta + 1)\, d\theta$

C. $\dfrac{1}{2}\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4\sin^2\theta - 4\sin\theta + 1)\, d\theta$

D. $\dfrac{1}{2}\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (8\sin^2\theta - 1 - 2\sin\theta)\, d\theta$

E. $\dfrac{1}{2}\displaystyle\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4\sin^2\theta + 4\sin\theta + 1)\, d\theta$

(10 pts) 22. The length of the curve $r = \sin^3\theta,\ 0 \leq \theta \leq \pi$ is:

A. $\displaystyle\int_0^{\pi} \sin\theta\sqrt{\sin\theta + 3\cos\theta}\, d\theta$

B. $\displaystyle\int_0^{\pi} \sin\theta\sqrt{\sin\theta + 9\sin^2\theta\cos^2\theta}\, d\theta$

C. $\displaystyle\int_0^{\pi} \sin\theta\sqrt{\sin^4\theta + 3\cos\theta}\, d\theta$

D. $\displaystyle\int_0^{\pi} \sin^2\theta\sqrt{1 + 8\cos^2\theta}\, d\theta$

E. $\displaystyle\int_0^{\pi} \sin^2\theta\sqrt{\sin^2\theta - 9\cos^2\theta}\, d\theta$

(8 pts) 23. Find a vertex of the conic section whose equation is $x^2 + 4x - 4y + 8 = 0$.

A. $(2, 1)$

B. $(2, -1)$

C. $(-2, 1)$

D. $(-2, -1)$

E. $(-1, 2)$

(8 pts) 24. The polar equation $r = \dfrac{5}{2 - 3\sin\theta}$ describes a conic section. The type of conic section and the directrix are:

A. ellipse, $y = \dfrac{5}{3}$

B. ellipse $y = -\dfrac{5}{3}$

C. hyperbola, $y = \dfrac{5}{3}$

D. hyperbola, $y = -\dfrac{5}{3}$

E. parabola, $y = \dfrac{5}{3}$