MA 16200
EXAM 3 Green
April 11, 2019

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Write the following in the TEST/QUIZ NUMBER boxes: **00** (and blacken in the appropriate digits below the boxes)
You must use a #2 pencil on the mark–sense sheet (answer sheet).On the mark–sense sheet, fill in your TA's name and the COURSE number. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark–sense sheet.
There are **12** questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the mark–sense sheet and the exam booklet when you are finished.
If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. Which statement is true about these two series?

$$\sum_{n=1}^{\infty} \frac{(-1.5)^n}{n^3} \quad \text{and} \quad \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

A. Both are conditionally convergent.

B. One is divergent and one is absolutely convergent.

C. Both are absolutely convergent.

D. One is absolutely convergent and one is conditionally convergent.

E. One is conditionally convergent and one is is divergent.

2. Suppose $b_n > 0$ for each n, and

$$\lim_{n\to\infty} \frac{b_{n+1}}{b_n} = \frac{1}{3}.$$

We know that the series $\displaystyle\sum_{n=0}^{\infty} (-1)^n b_n$

A. must be absolutely convergent.

B. must be divergent.

C. must be conditionally convergent.

D. converges or diverges; the given information is inconclusive.

E. converges; either conditional or absolute convergence can occur.

3. How many of these series converge?

$$\sum_{k=1}^{\infty} \frac{k!}{(k+2)!} \quad , \quad \sum_{\ell=0}^{\infty} \frac{1}{\ell + 2^{\ell}} \quad , \quad \sum_{m=1}^{\infty} \frac{m!}{m^m} \quad , \quad \sum_{n=1}^{\infty} \frac{n^3 3^n}{2^{2n}}$$

A. only two

B. all four

C. only one

D. only three

E. none

4. According to the Alternating Series Estimation Theorem, for which n is the following true?

$$\left| \ln 2 - \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots + \frac{(-1)^{n+1}}{n} \right) \right| \leq 0.01$$

Hint: $\ln(1+x) = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ with interval of convergence $(-1, 1]$

A. $n \geq 10$

B. $n \geq 27$

C. $n \geq 99$

D. $n \geq 11$

E. $n \geq 9$

5. Which of these x values is in the interval of convergence of the following power series?

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x+2)^n}{3n+1}$$

A. $x=-1$

B. $x=0$

C. $x=1$

D. $x=-3$

E. $x=2$

6. Find the Maclaurin series for

$$f(x)=\frac{3x^2}{(1+x^3)^2}$$

Hint: f is the derivative of a function with an easy Maclaurin formula

A. $\displaystyle\sum_{n=2}^{\infty}(-1)^n 3x^{6n}$

B. $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}\frac{x^{3n+1}}{3n+1}$

C. $\displaystyle\sum_{n=1}^{\infty}(-1)^n nx^{3n-3}$

D. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}3nx^{3n-1}$

E. $\displaystyle\sum_{n=0}^{\infty}(-1)^{n+1}x^{3n}$

7. Find the first few terms of the Taylor series centered at $a = \dfrac{1}{4}$ for the function $f(x) = \sqrt{x}$

A. $\dfrac{1}{2} + \dfrac{1}{2}\left(x - \dfrac{1}{4}\right) - \dfrac{1}{4}\left(x - \dfrac{1}{4}\right)^2 + \dfrac{3}{8}\left(x - \dfrac{1}{4}\right)^3 - \ldots$

B. $\dfrac{1}{2} + \dfrac{1}{2}\left(x - \dfrac{1}{4}\right) - \dfrac{1}{8}\left(x - \dfrac{1}{4}\right)^2 + \dfrac{1}{16}\left(x - \dfrac{1}{4}\right)^3 - \ldots$

C. $\dfrac{1}{2} + \left(x - \dfrac{1}{4}\right) - 2\left(x - \dfrac{1}{4}\right)^2 + 12\left(x - \dfrac{1}{4}\right)^3 - \ldots$

D. $\dfrac{1}{2} + \left(x - \dfrac{1}{4}\right) - \dfrac{1}{2}\left(x - \dfrac{1}{4}\right)^2 + \dfrac{1}{6}\left(x - \dfrac{1}{4}\right)^3 - \ldots$

E. $\dfrac{1}{2} + \left(x - \dfrac{1}{4}\right) - \left(x - \dfrac{1}{4}\right)^2 + 2\left(x - \dfrac{1}{4}\right)^3 - \ldots$

8. Use a Maclaurin series to choose the best estimate of

$$\int_0^{0.1} \sin(x^2)\, dx$$

Hint: $\cos x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ and $\sin x = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ for all x

A. $\dfrac{(0.1)^3}{3} - \dfrac{(0.1)^7}{42}$

B. $\dfrac{(0.1)^4}{2} - \dfrac{(0.1)^8}{24}$

C. $\dfrac{(0.1)^2}{2} - \dfrac{(0.1)^4}{24}$

D. $(0.1)^2 - \dfrac{(0.1)^6}{6}$

E. $(0.1)^3 - \dfrac{(0.1)^5}{6}$

9. Find the Maclaurin series for $f(x) = \dfrac{1}{\sqrt{1+x}}$.

A. $1 - \dfrac{1}{2}x + \displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n n!} x^n$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n+1)}{2^n n!} x^n$

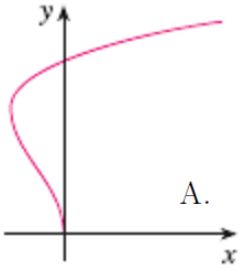
C. $1 + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n-1)(2n-3)}{2^n} x^n$

D. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)}{2^n} x^n$

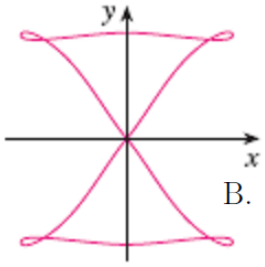
E. $1 + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n n!} x^n$

10. Match the parametric equations with the correct graph.

$$x = \frac{\sin 2t}{4+t^2}, \quad y = \frac{\cos 2t}{4+t^2}$$



A.



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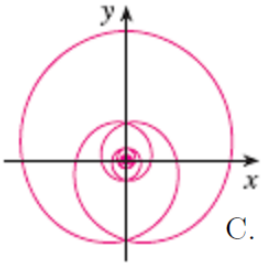

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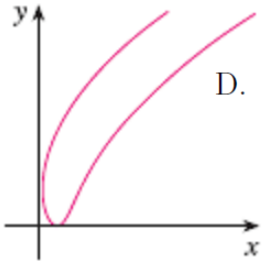
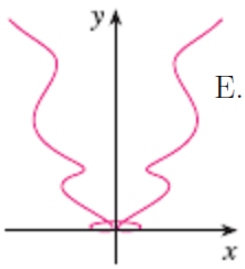


D.



E.

11. Consider the parametric equations

$$x = 2t^3 + 3t^2 - 12t, \quad y = 2t^3 + 3t^2 + 2.$$

Find a point (x, y) where the tangent is vertical.

A. $(52, -2)$

B. $(-7, 7)$

C. $(13, 3)$

D. $(4, 30)$

E. $(0, 2)$

12. A curve C is defined by the parametric equations $x = t^3 - 12t, \quad y = t^2 - 1$. Find all the values of t for which C is concave down.

A. $t < -2$ and $t > 2$

B. $t < -2$

C. all real t

D. $-2 < t < 2$

E. $t > 2$