MA162 — EXAM III — SPRING 2017 — APRIL 11, 2017
TEST NUMBER 01

INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 8 different test pages (including this cover page).
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. There are 14 problems and the number of points each problem is worth is indicated next to the problem number. The maximum possible score is 100 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have doubts, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
5. Do not consult notes or books.
6. **Do not handle** phones or cameras, calculators or any electronic device until after you have finished your exam, given it to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ____________________

STUDENT SIGNATURE: ____________________

STUDENT ID NUMBER: ____________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ____________________

FORMULA SHEET

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}, \quad \text{for all } x.$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}, \quad \text{for all } x.$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n, \text{ for all } x.$$

$$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \ldots \frac{k(k-1)(k-2)\ldots(k-n+1)}{n!} x^n \ldots$$
$$\text{for } |x| < 1.$$

If $f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n$, for $|x-a| < R$, then $\int_a^x f(t)\, dt = \sum_{n=0}^{\infty} \frac{C_n}{n+1} (x-a)^{n+1}$ for $|x-a| < R$.

If $f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n$, for $|x-a| < R$, then $f'(x)\, dt = \sum_{n=1}^{\infty} n C_n (x-a)^{n-1}$ for $|x-a| < R$.

1. (8 points) Consider the two series

$$I)\ \sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+7} \text{ and } II)\ \sum_{n=1}^{\infty}(-1)^n\frac{n}{5^n}.$$

Which of the following is true?

A. I and II converge conditionally

B. I converges conditionally and II converges absolutely

C. I converges absolutely and II converges conditionally

D. I and II converge absolutely

E. I and II diverge

2. (8 points) The series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(\ln(2n))^2}$

A. Converges by the ratio test

B. Diverges by the ratio test

C. Diverges by the integral test

D. Converges by the integral test

E. Converges by the root test.

3. (8 points) Which of the following series are convergent?

I. $\sum_{n=1}^{\infty} \dfrac{3n^2}{n^3+1}$

II. $\sum_{n=1}^{\infty} \dfrac{3n^2}{(n^3+1)^2}$

III. $\sum_{n=1}^{\infty} \dfrac{3n^2}{(n^3+1)^{4/3}}$

A. II only

B. I and II only

C. II and III only

D. None of them

E. All of them

4. (8 points) For what values of a is the series $\sum_{n=1}^{\infty} \left(\dfrac{n}{an+1}\right)^n$ absolutely convergent?

A. $|a| \leq 1$

B. $|a| < 1$

C. $|a| \geq 1$

D. $|a| > 1$

E. $|a| = 1$

5. (8 points) Compute the Taylor series of $f(x) = \ln x$ centered at 4 and use it to find $f^{(9)}(4)$.

A. $f^{(9)}(4) = \dfrac{9!}{4^9}$

B. $f^{(9)}(4) = -\dfrac{9!}{4^9}$

C. $f^{(9)}(4) = \dfrac{8!}{4^9}$

D. $f^{(9)}(4) = -\dfrac{8!}{4^9}$

E. $f^{(9)}(4) = \dfrac{10!}{4^9}$

6. (8 points) Using a geometric series, the first two nonzero terms of a power series for $\dfrac{x}{9-x^2}$ are

A. $\dfrac{x}{9} + \dfrac{x^3}{9}$

B. $\dfrac{x}{9} - \dfrac{x^2}{27}$

C. $\dfrac{x}{9} + \dfrac{x^2}{27}$

D. $\dfrac{x}{9} - \dfrac{x^3}{81}$

E. $\dfrac{x}{9} + \dfrac{x^3}{81}$

7. (8 points) Find the interval of convergence for $\sum_{n=2}^{\infty}(-4)^n \frac{x^n}{2n\sqrt{\ln n}}$.

A. $(-\frac{1}{2}, \frac{1}{2}]$

B. $[-\frac{1}{2}, \frac{1}{2})$

C. $(-\frac{1}{4}, \frac{1}{4}]$

D. $[-\frac{1}{4}, \frac{1}{4})$

E. $(-4, 4]$

8. (8 points) Find the Taylor series of $f(x) = \frac{1}{x^2+4x+6}$ centered at -2 and its radius of convergence. Notice that $x^2 + 4x + 6 = (x+2)^2 + 2$.

A. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x+2)^n$, with radius $R = 2$

B. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}(x+2)^n$, with radius $R = \sqrt{2}$

C. $f(x) = \sum_{n=0}^{\infty} \frac{1}{2^{n+1}}(x+2)^{2n}$, with radius $R = \sqrt{2}$

D. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x+2)^{2n}$, with radius $R = \sqrt{2}$

E. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(x+2)^{2n}$, with radius $R = 2$

9. (8 points) Use the root test to find the radius of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{(1+\frac{1}{n})^{n^2}}$.

A. e

B. 1

C. $\dfrac{1}{e}$

D. e^2

E. $\dfrac{1}{e^2}$

10. (8 points) Let $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(x-3)^n$ and let $F(x) = \displaystyle\int_3^x f(t)\,dt$. Which of the following gives an approximation of the value of $F(3.1)$ with an error less than or equal to 10^{-6}?

A. $F(3.1) \sim \dfrac{1}{2(10)^2} - \dfrac{1}{6(10)^3} + \dfrac{1}{12(10)^4}$

B. $F(3.1) \sim \dfrac{1}{2(10)^2} - \dfrac{1}{6(10)^3} + \dfrac{1}{14(10)^4}$

C. $F(3.1) \sim \dfrac{1}{2(10)^2} - \dfrac{1}{8(10)^3} + \dfrac{1}{15(10)^4}$

D. $F(3.1) \sim \dfrac{1}{2(10)^2} - \dfrac{1}{8(10)^3} + \dfrac{1}{16(10)^4}$

E. $F(3.1) \sim \dfrac{1}{2(10)^2} - \dfrac{1}{4(10)^3} + \dfrac{1}{6(10)^4}$

11. (8 points) The first three terms of the Maclaurin expansion of $f(x) = \dfrac{\sin(x^2) - x^2}{x^6}$ are:

A. $-\frac{1}{3!} + \frac{1}{7!}x^4 - \frac{1}{9!}x^8$

B. $-\frac{1}{3!} + \frac{1}{9!}x^4 - \frac{1}{6!}x^8$

C. $-\frac{1}{3!} + \frac{1}{7!}x^4 - \frac{1}{5!}x^8$

D. $-\frac{1}{3!} + \frac{1}{5!}x^4 - \frac{1}{9!}x^8$

E. $-\frac{1}{3!} + \frac{1}{5!}x^4 - \frac{1}{7!}x^8$

12. (4 points) If a series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n}$ also always converges absolutely.

A. True
B. False

13. (4 points) If $a_n \geq 0$ and the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} \sqrt{a_n}$ also always converges.

A. True
B. False

14. (4 points) The series $\displaystyle\sum_{n=1}^{\infty} (-1)^n e^{\frac{1}{n}}$ converges conditionally.

A. True
B. False