MATH 162 – SPRING 2004 – THIRD EXAM
APRIL 15, 2004

STUDENT NAME——————————————————

STUDENT ID——————————————————

RECITATION INSTRUCTOR——————————————

RECITATION TIME——————————————————

INSTRUCTIONS

1. Fill in your name, your student ID number, and your recitation instructor's name and recitation time above. Write your name, your student ID number and division and section number of your recitation section on your answer sheet, and fill in the corresponding circles.
2. Mark the letter of your response for each question on the mark-sense sheet.
3. There are 10 questions, each worth 10 points.
8. No books, notes or calculators may be used.

Useful formulas:

$$\text{Arc length} \quad L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\ \ dt$$

$$\text{Area of a surface of revolution} \quad S = \int_a^b 2\pi y(t)\sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

Some power series:

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \qquad \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \text{provided} \quad |x| < 1,$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n.$$

1) Find which series equals the definite integral $\int_0^1 \sin(x^2)\,dx$

A) $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+2)!}$

B) $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+3)!}$

C) $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+1)!(4n+3)}$

D) $\sum_{n=0}^{\infty}(-1)^{n-1} \frac{1}{(2n+5)!}$

E) $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+1)!(4n+2)}$

2) The power series expansion of $\frac{1}{(1+x)^2}$ is

A) $\sum_{n=0}^{\infty}(-1)^n x^n$

B) $\sum_{n=0}^{\infty}(-1)^n n x^{n-1}$

C) $\sum_{n=0}^{\infty}(-1)^{n-1} n x^{n-1}$

D) $\sum_{n=0}^{\infty}(-1)^{n-1} x^n$

E) $\sum_{n=0}^{\infty} x^n$

3) If $(1+x)^{1/3} = c_1 + c_2 x + c_3 x^2 + ...$ then c_3 is equal to

A) $\frac{1}{3}$

B) $\frac{1}{5}$

C) $\frac{1}{9}$

D) $\frac{1}{12}$

E) $-\frac{1}{9}$

4) The MacLaurin series of $x\cos(2x)$ is

A) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!}$

B) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{2n+1}}{(2n)!}$

C) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^{2n+1}}{(2n)!}$

D) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{n+1} x^{2n+1}}{(2n)!}$

E) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^{2n+1}}{(2n)!}$

5) The Taylor polynomial $T_2(x)$ for $f(x) = \sin x$ at $a = \frac{\pi}{3}$ is

A) $\frac{\sqrt{3}}{2} + \frac{1}{2}(x - \frac{\pi}{3}) - \frac{\sqrt{3}}{4}(x - \frac{\pi}{3})^2$

B) $\frac{\sqrt{3}}{2} + \frac{1}{2}(x - \frac{\pi}{3}) + \frac{\sqrt{3}}{4}(x - \frac{\pi}{3})^2$

C) $\frac{1}{2} - \frac{\sqrt{3}}{2}(x - \frac{\pi}{3}) - \frac{1}{4}(x - \frac{\pi}{3})^2$

D) $\frac{1}{2} + \frac{\sqrt{3}}{2}(x - \frac{\pi}{3}) + \frac{1}{4}(x - \frac{\pi}{3})^2$

E) $(x - \frac{\pi}{3}) - \frac{1}{6}(x - \frac{\pi}{3})^2$

6) The slope of the tangent line to the graph of the curve $x = 1 + t^2$, $y = t \ln t$ at $t = 2$ is

A) $\frac{1}{4}$

B) $\frac{\ln 2}{4}$

C) $\frac{4}{\ln 2}$

D) $\frac{1+\ln 2}{4}$

E) $\frac{4}{1+\ln 2}$

7) The length of the curve $x = e^t + e^{-t}$, $y = 2t$, $0 \leq t \leq 1$ is

A) $e + e^{-1} - 2$

B) $e - e^{-1}$

C) $e + e^{-1}$

D) $e + e^{-1} - 2$

E) $\frac{1}{2}(e + e^{-1})$

8) The curve $x = \cos^3\theta$, $y = \sin^3\theta$, $0 \leq \theta \leq \frac{\pi}{2}$ is rotated about the x-axis to generate a surface. Its area is given by

A) $\int_0^{\frac{\pi}{2}} 6\pi \cos\theta \sin\theta \, d\theta$

B) $\int_0^{\frac{\pi}{2}} 6\pi \cos^2\theta \sin^2\theta \, d\theta$

C) $\int_0^{\frac{\pi}{2}} 6\pi \cos^2\theta \sin^3\theta \, d\theta$

D) $\int_0^{\frac{\pi}{2}} 6\pi \cos\theta \sin^4\theta \, d\theta$

E) $\int_0^{\frac{\pi}{2}} 6\pi \cos^2\theta \sin^4\theta \, d\theta$

9) The cartesian coordinates of a point are $(-2\sqrt{3}, 2)$. Find its polar coordinates

A) $(4, \frac{2\pi}{3})$

B) $(4, \frac{5\pi}{6})$

C) $(2, \frac{2\pi}{3})$

D) $(2, \frac{5\pi}{6})$

E) $(4, -\frac{\pi}{3})$

10) The polar equation of the circle of radius 1 centered at $(0, -1)$ is

A) $r = 2\cos\theta$

B) $r = 2\sin\theta$

C) $r = -\sin\theta$

D) $r = -2\sin\theta$

E) $r = -2\cos\theta$