MA 162 **EXAM 3** **Spring, 2003**

Name:_____ **ID:**_____

Recitation Instructor:_____ **Recitation Time:**_____

Instructions: Give a complete answer to problems 13 – 18 in the space provided, if necessary. Be sure to **show all your work**. Answers not supported by work will receive little credit. **Write the answer to each question in the box provided.** Write your name and ID number on each page of the exam. Also write your Recitation Instructor's name and Recitation time above. No books, notes or calculators may be used on this exam. This exam has 5 pages, including this page.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \quad |x| < \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \quad |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, \quad |x| < 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \quad |x| < \infty$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n, \quad |x| < 1$$

Taylor series of $f(x)$ at a:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

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Write T or F before each problem number on the line provided to indicate whether each statement is True or False. The terms a_n and b_n are positive. Each of these true–false questions are worth 4 points.

_____ **1.** If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.

_____ **2.** If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

_____ **3.** If $\lim_{n\to\infty} \left(\frac{a_{n+1}}{a_n}\right) = \frac{1}{2}$ then $\sum_{n=1}^{\infty} a_n$ diverges.

_____ **4.** If $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$ then $\sum_{n=1}^{\infty} a_n$ converges.

_____ **5.** If $\lim_{n\to\infty} \frac{a_n}{\left(\frac{1}{n^2}\right)} = 2$ then $\sum_{n=1}^{\infty} a_n$ converges.

_____ **6.** If $b_n < a_n$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} b_n$ converges.

_____ **7.** The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges conditionally.

_____ **8.** If $f(x) = 2 - x + x^2 - x^3 + x^4 - \cdots$, then $f''(0) > 0$.

_____ **9.** If $f(x) = \sum_{n=1}^{\infty} \frac{(n+1)!}{(2n)!} x^n$ then $f'''(0) = \frac{1}{5}$.

_____ **10.** If $f(x) = \sum_{n=1}^{\infty} a_n (x-a)^n$ then $a_n = f^{(n)}(a)$.

_____ **11.** The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges absolutely.

_____ **12.** The radius of convergence of the series $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$ is $\frac{1}{2}$.

Name:_____ ID:_____

13. (8 points) Consider the convergent alternating series $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$.

a.) Write out the first 6 terms of the series.

b.) Find the smallest number of terms which, when added, estimate the sum of the series with error < 0.1.

The smallest number of terms is

14. (8 points) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+n}}$ is absolutely convergent, conditionally convergent or divergent. You must show all necessary work and circle your conclusion in the smaller box.

Show all necesary work here:

The series is absolutely convergent, conditionally convergent, divergent

Name:_____ ID:_____

15. (8 points) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}$.

Don't forget to check for convergence at the endpoints of the interval. You must show all your work.

Interval of convergence is:

16. (10 points) Find a formula for the n–th term (a_n) of the Maclaurin Series for $f(x) = \frac{1}{(1+x)^3}$.

$a_n =$

Name:_____ ID:_____

17. **(10 points)** Find the 2nd degree Taylor polynomial approximation ($T_2(x)$) to $f(x) = \tan x$ at $a = \pi/4$.

$T_2(x) =$

18. **(8 points)** Sketch on the axes provided below the curve $x = 5\sin t$, $y = 2\cos t$, for $0 \leq t \leq \pi$. Indicate the path's direction for increasing t with arrowheads on the graph.