MATH 162 – FALL 2007 – THIRD EXAM
NOVEMBER 15, 2007

STUDENT NAME________________________

STUDENT ID________________________

INSTRUCTOR ________________________

RECITATION INSTRUCTOR________________________

RECITATION TIME________________________

INSTRUCTIONS

1. Verify that you have **7 pages.**
2. The exam has twelve questions, each worth 8.3 points.
3. Fill in the blank spaces above.
4. Use a number 2 pencil to write on your **mark-sense sheet**.
5. **On your mark sense sheet,** write your name, your student ID number, the division and section numbers of your recitation, and fill the corresponding circles.
6. Mark the letter of your response for each question on this booklet and on the mark-sense sheet.
7. Work only on the spaces provided or on the backside of the pages.
8. **No books, notes or calculators may be used.**

1.A 2.B 3.B 4.E 5.C 6. C

7.A 8.C 9.A 10.D 11.B 12.C

1)(8.3 points) The series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2+1}$ converges absolutely

A) True

B) False

2)(8.3 points) Which of the following series converge?

$$I)\sum_{n=1}^{\infty}\frac{1}{n}, \quad II)\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n, \quad III)\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n$$

A) only I

B) only II

C) only III

D) only I and II

E) only II and III

3)(8.3 points) Suppose we know that $a_n \geq \frac{1}{2n}$, $n = 1, 2, 3, \ldots$ Which statement below must be true?

A) $\sum_{n=1}^{\infty} a_n$ converges

B) $\sum_{n=1}^{\infty} a_n$ diverges

C) $\sum_{n=1}^{\infty} a_n$ converges, provided $\lim_{n\to\infty} a_n = 0$

D) $\sum_{n=1}^{\infty} a_n$ converges, provided $a_n \geq a_{n+1}$ for all n

E) None of the statements above is necessarily true

4)(8.3 points) Suppose we want to approximate the sum of the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^3}$ by the sum $s_m = \sum_{n=1}^{m}(-1)^n \frac{1}{n^3}$ of the first m terms. By the theory of alternating series, the error will be less than 10^{-3} provided $m =$

A) 4

B) 5

C) 6

D) 7

E) 10

5)(8.3 points) The series

$$\sum_{k=1}^{\infty} \frac{5^k k^k}{(2k-1)^{2k}} \text{ is}$$

A) convergent because $\lim_{k\to\infty} \frac{5^k k^k}{(2k-1)^{2k}} = 0$

B) divergent because $\lim_{k\to\infty} \frac{5^k k^k}{(2k-1)^{2k}}$ is not equal to zero

C) convergent by the root test

D) divergent by the root test

E) divergent by the ratio test

6)(8.3 points) The series

$$\sum_{m=1}^{\infty} \frac{m!}{4^{2m} m^4} \quad \text{is}$$

A) convergent by the root test

B) convergent by the integral test

C) divergent by the ratio test

D) convergent by the ratio test

E) none of the alternatives above is correct

7)(8.3 points) Which of the following series converge?

$$I)\ \sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2} \qquad\qquad II)\ \sum_{k=2}^{\infty} \frac{1}{k(\ln k)^{\frac{1}{3}}}$$

A) only I

B) only II

C) neither

D) both

E) I converges conditionally and II converges absolutely

8)(8.3 points) The interval of convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^n}{4n\ 3^n} \qquad \text{is}$$

A) $[-2, 4]$

B) $(-3, 3)$

C) $(-2, 4]$

D) $[-2, 4)$

E) $(-3, 3]$

9)(8.3 points) Which of the following is a power series representation of the function
$$f(x) = \frac{1}{x^2 - 2x + 2}\ ?$$

A) $\sum_{n=0}^{\infty}(-1)^n(x-1)^{2n}$

B) $\sum_{n=0}^{\infty}(x-1)^{2n}$

C) $\sum_{n=0}^{\infty}(-1)^n\frac{(x-1)^n}{3n+1}$

D) $\sum_{n=0}^{\infty}(-1)^n\frac{(x-1)^n}{n^2}$

E) $\sum_{n=0}^{\infty}(-1)^n(x-1)^n$

10)(8.3 points) Let $f(x)$ be the function which is represented by the power series
$$f(x) = \sum_{n=1}^{\infty}(-1)^n\frac{(x-4)^n}{2\,n^3}.$$
The third derivative of the function f at $x = 4$ is equal to

A) 1/9

B) −2/3

C) 1/27

D) −1/9

E) 1/25

11)(8.3 points) Let $f(x)$ be a function such that $f'(x) = x^2 \cos x$ and that $f(0) = 0$. The Maclaurin series of $f(x)$ is

A) $\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{(2n+3)(2n)!}$

B) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+3}}{(2n+3)(2n)!}$

C) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+4}}{(2n+5)(2n)!}$

D) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{n+2}}{(2n+1)!}$

E) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{3n+1}}{(2n+3)!}$

12)(8.3 points) The first three terms of the binomial series expansion of

$$f(x) = (1+2x)^{-\frac{1}{4}}$$

are

A) $1 - \frac{1}{3}x + \frac{5}{9}x^2$

B) $1 - \frac{1}{4}x + \frac{5}{32}x^2$

C) $1 - \frac{1}{2}x + \frac{5}{8}x^2$

D) $1 - \frac{1}{5}x + \frac{5}{9}x^2$

E) $1 - \frac{1}{2}x + \frac{3}{25}x^2$