MA 162 **EXAM 3** **Fall 2002**

Name _____ Student ID number _____

Lecturer _____ Recitation Instructor _____

Instructions:

1. This package contains 12 problems, for a total of 100 points.
2. Please supply all information requested above and on the mark–sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly.
4. No books, notes, or calculators, please.

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

(12 pts) 1. The interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(2x-5)^n}{n3^n}$ is:

A. (∞, ∞)

B. $(-3, 3)$

C. $[1, 4)$

D. $(1, 4)$

E. $[-3, 3)$

(4 pts) 2. Let $a_n = \frac{3n^3 - 4n^2 + 5}{6n^5 - 2n + 7}$ and $b_n = \frac{1}{n^3}$. Then the limit comparison test with $\sum_{n=1}^{\infty} b_n$ shows that $\sum_{n=1}^{\infty} a_n$ converges.

A. True

B. False

(10 pts) 3. The series $\sum_{n=0}^{\infty} n^2 e^{-n^3}$ is:

A. Divergent by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$

B. Divergent by comparison with $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

C. Divergent by the integral test

D. Convergent by the integral test

E. Convergent by comparison with $\sum_{n=0}^{\infty} e^{-n^2}$

(6 pts) 4. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{3+2n}$ is:

A. Divergent

B. Absolutely convergent

C. Conditionally convergent

(6 pts) 5. If we wish to approximate $s = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^3}$ by a partial sum s_k to within 10^{-6}, the error estimates say we must choose k to be at least:

A. 20

B. 10

C. 100

D. 75

E. 50

(12 pts) 6. Which of the following series converge?

I. $\sum_{n=0}^{\infty} \frac{3^{2n}}{(\ln n)^n}$

II. $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{2n-3}$

III. $\sum_{n=0}^{\infty} \frac{4^{2n}(x+2)^{2n}}{\sqrt{n+1}}$, for $\frac{3}{2} < x < \frac{5}{2}$

A. I only

B. I and II

C. I and III

D. III only

E. None of the series converges

(8 pts) 7. Find the power series expansion for the function $\frac{d}{dx}\left(\frac{1}{1+x^2}\right)$.

A. $\sum_{n=1}^{\infty} nx^{n-1}$

B. $\sum_{n=1}^{\infty} (-1)^n nx^{n-1}$

C. $\sum_{n=1}^{\infty} 2nx^{2n-1}$

D. $\sum_{n=1}^{\infty} (-1)^n 2nx^{2n-1}$

E. $\sum_{n=1}^{\infty} (-1)^n (2n+1)x^{2n+1}$

(8 pts) 8. Represent $\int_0^1 \cos(x^2)dx$ as an infinite series.

A. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!}$

B. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!}$

C. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(4n)!}$

D. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(4n+1)!}$

E. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!(4n+1)}$

(8 pts) 9. If $\ln(x)$ is expanded as a power series of the form $\sum_{n=0}^{\infty} c_n(x-3)^n$, then $c_3 =$

A. $\frac{1}{6}$

B. $\frac{1}{24}$

C. $-\frac{1}{18}$

D. $\frac{1}{54}$

E. $\frac{1}{81}$

(8 pts) 10. Find the first three terms of the Maclaurin series of $f(x) = \sqrt[3]{8+4x}$.

A. $2+\frac{x}{3}-\frac{1}{9}x^2$

B. $2+\frac{x}{3}-\frac{1}{18}x^2$

C. $2+\frac{2x}{3}-\frac{2}{9}x^2$

D. $1+\frac{x}{3}-\frac{1}{9}x^2$

E. $1+\frac{2x}{3}-\frac{2}{9}x^2$

(8 pts) 11. Find the slope of the tangent line to the curve parametrized by $x = te^t$, $y = t\ln(t)$, at $t = 1$.

A. e

B. $2e$

C. $\frac{1}{e}$

D. $\frac{1}{2e}$

E. 1

(10 pts) 12. Find the length of the curve $x = 3t^2$, $y = 2t^3$, $0 \leq t \leq 1$.

A. $2(2\sqrt{2} - 1)$

B. $4(2\sqrt{2} - 1)$

C. $2\sqrt{2} - 1$

D. $3(2\sqrt{2} - 1)$

E. $6(2\sqrt{2} - 1)$