NAME_____

STUDENT ID_____

RECITATION INSTRUCTOR_____

RECITATION TIME_____

1. Fill in your name, your student ID, your recitation instructors name, and your recitation time above.

2. On the answer sheet, write your **name**, your **division and section number**, your **student identification number**, and the **test number**. Fill in the corresponding circles.

3. There are 11 questions. The first 10 are worth 9 points each and the last is worth 10 points. For each question, mark the letter corresponding to your answer on the answer sheet.

4. At the end of the exam turn in both the question sheets and the answer sheet.

5. No books, notes, or calculators may be used.

1. Which of the following integrals arises when one makes a suitable trigonometric substitution to compute

$$\int \frac{x^2}{\sqrt{4-x^2}}\,dx.$$

A. $\displaystyle\int 4\sin^2\theta\,d\theta$

B. $\displaystyle\int \frac{2\sin^2\theta}{\cos\theta}\,d\theta$

C. $\displaystyle\int \frac{\tan^2\theta\sec\theta}{4}\,d\theta$

D. $\displaystyle\int \frac{\tan^2\theta}{4\sec\theta}\,d\theta$

E. $\displaystyle\int \frac{\sin^2\theta}{4\cos^2\theta}\,d\theta$

2. Compute $\displaystyle\int_2^4 \frac{dx}{\sqrt{x^2-4}}$.

A. $\ln(\sqrt{2}+2)$

B. $\frac{1}{2}\ln(2\sqrt{2}+3)$

C. $\sqrt{2}+\frac{1}{2}\ln(\sqrt{2}+1)$

D. $\ln(2+\sqrt{3})$

E. $2\ln(2\sqrt{2}+3)$

3. Evaluate $\displaystyle\int \frac{2x+5}{x^2+2x+2}\,dx.$

A. $3\ln|x^2+2x+2|+\tan^{-1}(x^2+2x+2)+C$

B. $\ln|x^2+2x+2|+3\tan^{-1}(x+1)+C$

C. $\ln|x^2+2x+2|+\dfrac{3}{x^2+2x+2}+C$

D. $2x+2+3\tan^{-1}(x+1)+C$

E. $2\ln|2x+2|+3\tan^{-1}(x+1)+C$

4. What is the form of the partial fraction decomposition of

$$\frac{x+2}{(x-1)^2(x+1)(x^2+4)^2}?$$

A. $\dfrac{A}{(x-1)^2}+\dfrac{B}{x+1}+\dfrac{Cx+D}{(x^2+4)^2}$

B. $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+1}+\dfrac{D}{x^2+4}+\dfrac{E}{(x^2+4)^2}$

C. $\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{Cx+D}{x^2+4}$

D. $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2-1}+\dfrac{Dx+E}{x^2+4}+\dfrac{Fx+G}{(x^2+4)^2}$

E. $\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+1}+\dfrac{Dx+E}{x^2+4}+\dfrac{Fx+G}{(x^2+4)^2}$

5. Evaluate $\displaystyle\int_0^1 \frac{4}{x^2+4x+3}\,dx.$

A. $\ln 2 + \ln 4 - \ln 3$

B. $\frac{1}{2}\ln 2 - \ln 4 + \frac{3}{2}\ln 3$

C. $\frac{1}{2}(\ln 2 - \ln 4 + 3\ln 3)$

D. $2(\ln 2 + 2\ln 4 - \ln 3)$

E. $2(\ln 2 - \ln 4 + \ln 3)$

6. Evaluate $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}+1}$.

A. $1+\ln 2$

B. $2-4\ln 2$

C. $\frac{1}{2}+2\ln 2$

D. $2-2\ln 2$

E. $2+\frac{1}{2}\ln 2$

7. Find the length of the curve $f(x)=\ln(\sec x),\ 0\le x\le \pi/3$.

A. $\ln(1+\sqrt{2})$

B. $\sqrt{2}+\sqrt{3}$

C. $\ln(\sqrt{2}+\sqrt{3})$

D. $1+\sqrt{2}$

E. $\ln(2+\sqrt{3})$

8. A surface is generated by rotating the curve $y = 2\sqrt{1+x}$, $0 \le x \le 2$, about the x–axis. Find the surface area of the surface.

A. $\frac{8\pi}{3}(2-\sqrt{2})$

B. $\frac{4\pi}{3}(\sqrt{3}-\sqrt{2})$

C. $\frac{8\pi}{3}(8-2^{3/2})$

D. $\frac{\pi}{8}(\sqrt{3}-1)$

E. $\frac{8\pi}{3}(8-\sqrt{2})$

9. A lamina of uniform density has the shape of the region bounded by

$$y = x, \quad \text{and} \quad y = x^4.$$

The area of the region is $\frac{3}{10}$. Which expression gives the y–coordinate $\bar{y}$ of the center of mass.

A. $\bar{y} = \frac{10}{3}\int_0^1 (x^2 - x^5)\,dx$

B. $\bar{y} = \frac{5}{3}\int_0^1 (x - x^4)^2\,dx$

C. $\bar{y} = \frac{10\pi}{3}\int_0^1 (x^2 - x^5)\,dx$

D. $\bar{y} = \frac{5}{3}\int_0^1 (x^2 - x^8)\,dx$

E. $\bar{y} = \frac{10\pi}{3}\int_0^1 (x^2 - x^8)\,dx$

10. The curve $y = e^x$, $0 \le x \le 2$, is rotated about the y–axis. Which integral gives the surface area of the surface of revolution.

A. $\displaystyle\int_0^2 2\pi e^x \sqrt{1+e^{2x}}\,dx$

B. $\displaystyle\int_0^2 2\pi x \sqrt{1+e^{2x}}\,dx$

C. $\displaystyle\int_0^2 2\pi x e^x\,dx$

D. $\displaystyle\int_0^2 \pi x e^{2x} \sqrt{1+e^{2x}}\,dx$

E. $\displaystyle\int_0^2 \pi e^{2x}\,dx$

11. Which statement is true about the following improper integrals.

I. $\displaystyle\int_{-1}^1 \frac{1}{x}\,dx$ II. $\displaystyle\int_1^\infty \frac{1}{e^x}\,dx$ III. $\displaystyle\int_\pi^\infty \frac{\sin^2 x}{x^2}\,dx$

A. II and III converge. I diverges.

B. I and II converge. III diverges.

C. II converges. I and III diverge.

D. I, II and III converge.

E. I, II and III diverge.