MATH 162 – FALL 2006 – SECOND EXAM
OCTOBER 18, 2006

STUDENT NAME____

STUDENT ID____

RECITATION INSTRUCTOR____

RECITATION TIME____

INSTRUCTIONS

1. Verify that you have **8 pages.**
2. Fill in the blank spaces above.
3. Use a number 2 pencil to write on your **mark-sense sheet**.
4. **On your mark sense sheet**, write your name, your student ID number, the division and section numbers of your recitation, and fill the corresponding circles.
5. Mark the letter of your response for each question on the **mark-sense sheet.**
6. There are 13 questions. The **first eight problems are worth 5 points each. The other 5 are worth 12 points each**.
7. Show as much as possible of your work. Although this exam will be machine graded, in certain situations it may be necessary that we look at your exam.
8. No books, notes or calculators may be used.

USEFUL FORMULAS

Moments and center of mass

$$M_x = \int_a^b \frac{1}{2}\left((f(x))^2 - (g(x))^2\right) dx, \quad M_y = \int_a^b x\,(f(x) - g(x))\,dx$$

$$\overline{x} = \frac{M_y}{M}, \quad \overline{y} = \frac{M_x}{M},$$

Arc length

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\;\; dx$$

Area of a surface of revolution ABOUT THE x-AXIS

$$S = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx$$

1) (5 points) Indicate the technique most likely to produce an expression for which we may directly find an antiderivative of

$$\int x^2(16x^2-9)^{-\frac{1}{2}}\,dx$$

(**You do not need to compute the integral, just indicate how you would do it if you were asked to do so.**)

A) Integration by parts

B) Trigonometric substitution

C) Partial fractions

D) Direct substitution

E) None of the above

2) (5 points) Indicate the technique most likely to produce an expression for which we may directly find an antiderivative of

$$\int \frac{8x^2-2}{x^2-2x-8}\,dx$$

(**You do not need to compute the integral, just indicate how you would do it if you were asked to do so.**)

A) Integration by parts

B) Trigonometric substitution

C) Partial fractions

D) Direct substitution

E) None of the above

3) (5 points) Indicate the technique most likely to produce an expression for which we may directly find an antiderivative of

$$\int \frac{x^3}{\sqrt{x^4+8}}\,dx$$

(**You do not need to compute the integral, just indicate how you would do it if you were asked to do so.**)

A) Integration by parts

B) Trigonometric substitution

C) Partial fractions

D) Direct substitution

E) None of the above

4) (5 points) Indicate the technique most likely to produce an expression for which we may directly find an antiderivative of

$$\int x^3 \sin x\,dx$$

(**You do not need to compute the integral, just indicate how you would do it if you were asked to do so.**)

A) Integration by parts

B) Trigonometric substitution

C) Partial fractions

D) Direct substitution

E) None of the above

5)(5 points) If we are to find
$$\int \frac{3x+1}{(x+1)^2(x^2+1)}\,dx, \text{ we first write it in the form}$$
(**You do not need to compute the integral, just indicate how you would do it if you were asked to do so.**)

A) $\frac{A}{(x+1)^2} + \frac{B}{x^2+1}$

B) $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1}$

C) $\frac{A}{(x+1)^2} + \frac{Bx+C}{x^2+1}$

D) $\frac{Ax+B}{(x+1)} + \frac{C}{x^2+1}$

E) $\frac{A+B}{(x+1)^2} + \frac{C+D}{x^2+1}$

6) (5 points) The table of integrals on the back of the book says that
$$\int \frac{u\,du}{\sqrt{1+u}}\,du = \frac{2}{3}(u-2)\sqrt{1+u} + C$$
Based on this, we conclude that
$$\int_0^{\frac{3}{2}} \frac{u\,du}{\sqrt{1+2u}}\,du \text{ is equal to}$$
A) $\frac{1}{2}$

B) $\frac{3}{2}$

C) $\frac{2}{3}$

D) $\frac{2}{3}\sqrt{3}$

E) $3(1+\sqrt{3})$

7) (5 points) The midpoint approximation, with $n = 3$, to $\int_0^6 e^{-x^2}\, dx$ is

A) $2\left(1 + e^{-2} + e^{-9} + e^{-25}\right)$

B) $2\left(1 + e^{-1} + e^{-4}\right)$

C) $2\left(e^{-1} + e^{-9} + e^{-25}\right)$

D) $2\left(1 + e^{-4} + e^{-9} + e^{-25}\right)$

E) $\frac{1}{2}\left(e^{-1} + e^{-9} + e^{-25}\right)$

8)(5 points) Which of the following statements are true?

I) If the sequence $\{a_n\}$ converges, then $a_n \to 0$

II) If the series $\sum a_n$ converges, then $a_n \to 0$

III) If $a_n \to 0$, then the series $\sum a_n$ converges

A) I and II are true, III is false

B) I and III are true, II is false

C) Only I is true

D) Only II is true

E) Only III is true

9)(12 points) The length of the curve $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$, $1 \le x \le 2$ is (**see formulas on page one of the exam**)

A) $\frac{3}{2} + \frac{1}{4}\ln 2$

B) $\frac{1}{2} + \frac{1}{3}\ln 2$

C) $2 + \frac{1}{8}\ln 2$

D) $4 + 2\ln 2$

E) $1 + \ln 2$

10)(12 points) Consider the region in the first quadrant bounded by $y = x^2$, $x = 1$ and $x = 2$. The coordinates of its centroid $(\bar{x}, \bar{y})$ are (**see formulas on page one of the exam**)

A) $(\frac{25}{28}, \frac{1}{2})$

B) $(\frac{45}{28}, \frac{93}{35})$

C) $(\frac{45}{56}, \frac{93}{70})$

D) $(\frac{90}{28}, \frac{94}{73})$

E) $(\frac{45}{28}, \frac{93}{70})$

11)(12 points) The area of the surface obtained by rotating the curve $y = \frac{1}{3}x^3$, $0 \leq x \leq 1$ about the x-axis is (**see formulas on page one of the exam**)

A) $\frac{\pi}{3}$

B) $\frac{\pi}{9}$

C) $\frac{\pi}{3}(2\sqrt{2} - 1)$

D) $\frac{\pi}{9}(2\sqrt{2} - 1)$

E) $\frac{\pi}{9}(3\sqrt{2} - 1)$

12) (12 points) Evaluate

$$\int_0^{\sqrt{3}} \frac{x^3}{\sqrt{1 + x^2}}\, dx.$$

A) $\frac{1}{3}$

B) $\frac{2}{3}$

C) $\frac{4}{3}$

D) $\frac{5}{3}$

E) 2

13)(12 points) Use that $\lim_{x\to 0} x^p \ln x = 0$, for any $p > 0$, to compute the improper integral

$$\int_0^1 x^{-\frac{1}{2}} \ln x \, dx$$

A) -4

B) -3

C) -5

D) -2

E) -8