MA 16200
EXAM 1
September 21, 2021

NAME ____________________ YOUR TA'S NAME ____________________

STUDENT ID # ____________________ RECITATION TIME ____________________

Write the following in the TEST/QUIZ NUMBER boxes: **1010** (and blacken in the appropriate digits below the boxes)

You must use a #2 pencil on the answer sheet. On the scantron sheet, fill in your TA's name and the COURSE number, MA 162. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. Fill in your three-digit SECTION NUMBER. If you do not know your section number, ask your TA. Complete the signature line.

There are **12** questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron sheet and the exam booklet when you are finished.

If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____________________

STUDENT SIGNATURE: ____________________

1. The point $(2, 0, -3)$ lies on a sphere centered at $(1, 1, 1)$. What is the radius of the sphere?

A. 4

B. $\sqrt{14}$

C. $\sqrt{6}$

D. 18

E. $3\sqrt{2}$

F. 6

2. Which of these three-dimensional vectors is **not** a unit vector?

A. $\left\langle 0, \frac{4}{5}, \frac{3}{5} \right\rangle$

B. $\left\langle \frac{2}{7}, -\frac{3}{7}, \frac{6}{7} \right\rangle$

C. $\vec{\boldsymbol{\jmath}}$

D. $\frac{1}{\sqrt{3}}\left(\vec{\boldsymbol{\imath}} + \vec{\boldsymbol{\jmath}} + \vec{\boldsymbol{k}}\right)$

E. $\left\langle \cos\frac{\pi}{10}, 0, \sin\frac{\pi}{10} \right\rangle$

F. $\left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle$

3. $\vec{\boldsymbol{v}} = \langle -1, -2\rangle$ and $\vec{\boldsymbol{w}} = \langle 2, 3\rangle$. Find $\text{proj}_{\vec{\boldsymbol{v}}}\vec{\boldsymbol{w}}$, the vector projection of $\vec{\boldsymbol{w}}$ onto $\vec{\boldsymbol{v}}$.

A. $\left\langle \frac{8}{13}, \frac{16}{13} \right\rangle$

B. $\left\langle \frac{8}{5}, \frac{16}{5} \right\rangle$

C. $\Big\langle -2, -6 \Big\rangle$

D. $\left\langle \frac{4}{5}, \frac{8}{5} \right\rangle$

E. $\left\langle \frac{4}{13}, \frac{8}{13} \right\rangle$

F. $\left\langle -\frac{16}{\sqrt{13}}, -\frac{24}{\sqrt{13}} \right\rangle$

4. The three points $(0, 0, 0), (1, 1, 1)$, and $(a, 7, 5)$ form the vertices of a triangle with area $\frac{\sqrt{6}}{2}$. Select the value of a from the following choices:

A. $a = 6$

B. $a = 3$

C. $a = 4$

D. $a = 2$

E. $a = 12$

F. $a = 1$

5. Suppose $|\vec{\boldsymbol{v}} \times \vec{\boldsymbol{w}}| = 3$ and $\tan\theta = \sqrt{3}$, where θ is the angle between $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{w}}$. Find $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{w}}$.

A. 3

B. 1

C. $\sqrt{3}$

D. $3\sqrt{3}$

E. $\sqrt{3}/2$

F. $1/\sqrt{3}$

6. Find the area of the region bounded by the curves $y = e^x$, $y = e^{-x}$, and $x = 1$.

A. $e - 1/e$

B. $e + 1/e$

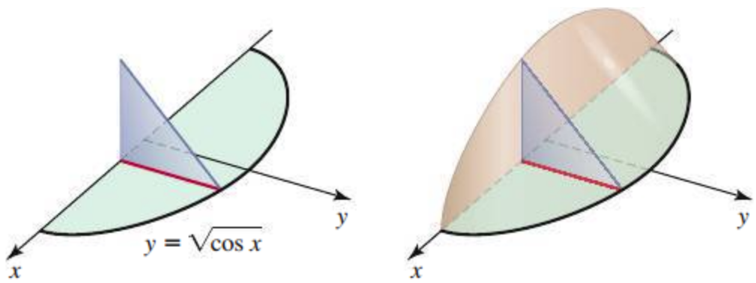
C. $\dfrac{(e+1)^2}{e}$

D. $\dfrac{(e-1)^2}{e}$

E. $1 - 1/e$

F. $1 + 1/e$

7. Find the volume of the solid whose base is the region bounded by $y = \sqrt{\cos x}$ and the x-axis on $[-\pi/2, \pi/2]$, and whose cross sections through the solid perpendicular to the x-axis are isosceles right triangles with a horizontal leg in the xy-plane and a vertical leg above the x-axis. (Hint: an isosceles right triangle has legs of equal length).



A. $\frac{1}{2}$

B. 1

C. 2

D. π

E. 2π

F. $\frac{\pi}{2}$

8. Find a formula for the volume of the following solid:
The region bounded by $y = \sqrt{x}$, the x-axis, and $x = 1$, is revolved about the line $y = 1$.

A. $\displaystyle\int_0^1 2\pi(1-y)(1-y^2)\,dy$

B. $\displaystyle\int_0^1 2\pi(1-x)(1-\sqrt{x})\,dx$

C. $\displaystyle\int_0^1 2\pi y(1-y^2)\,dy$

D. $\displaystyle\int_0^1 2\pi x(1-\sqrt{x})\,dx$

E. $\displaystyle\int_0^1 2\pi y^3\,dy$

F. $\displaystyle\int_0^1 2\pi x^{3/2}\,dx$

9. Find a function whose arc length on the interval $[a, b]$ is given by the following integral:

$$\int_a^b \sqrt{9x^4 + 6x^2 + 2}\, dx$$

A. $y = 9x^4 + 6x^2 + 1$

B. $y = 3x^2 + 1$

C. $y = 6x + 1$

D. $y = \dfrac{9}{5}x^5 + 2x^3 + x$

E. $y = x^3 + x$

F. $y = 3x^2 + \sqrt{6}x + 1$

10. Find the area of the surface obtained by rotating the curve $y = \dfrac{x^3}{3}$ about the x-axis, for $0 \le x \le \sqrt[4]{3}$.

A. $\pi \cdot 3^{-13/8}$

B. $\dfrac{\pi}{24}$

C. $\dfrac{7\pi}{9}$

D. $\dfrac{8\pi}{9}$

E. $\left(\sqrt[4]{3}\sqrt{1 + \sqrt{3}}\right)\pi$

F. $\left[\sqrt[4]{3}\sqrt{1 + \sqrt{3}} + \ln\left(\sqrt{1 + \sqrt{3}} + \sqrt[4]{3}\right)\right]\pi$

11. Find the volume of the solid generated when the region bounded by the following curves is revolved about the x-axis:
$y = x + 2$, $y = x$, $x = 0$, $x = 2$

A. 8π

B. 6π

C. 24π

D. 4π

E. 12π

F. 16π

12. A force of 40 N is holding a spring 0.1 m from its equilibrium position. How much work is needed to stretch the spring an **additional** 0.1 m, so that it is 0.2 m from equilibrium?

A. 2 N·m

B. 4 N·m

C. 6 N·m

D. 20 N·m

E. 40 N·m

F. 60 N·m