MA 16100
FINAL EXAM Form 01
May 4, 2022

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below the boxes on the scantron: **01**

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough space for your name, fill in as much as you can.
2. Section number. If you don't know your section number, ask your TA.
3. Test/Quiz number: **01**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros.**

There are **25** questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 9:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 9:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: _____

1. The domain of $f(x) = \sqrt{x-2} - \dfrac{1}{\sqrt{3-x}}$ is

A. $[2, 3)$

B. $(2, 3)$

C. $(2, 3]$

D. $[2, \infty)$

E. $[2, 3]$

2. Express the given quantity as a single logarithm

$$\frac{2}{5}\ln(x+7) - \ln\sqrt[5]{x} - \frac{1}{5}\ln(x^2-8)$$

A. $\ln\sqrt[5]{\dfrac{(x+7)^2(x^2-8)}{x}}$

B. $\ln\sqrt[5]{\dfrac{x(x+7)^2}{(x^2-8)}}$

C. $\ln\sqrt[5]{\dfrac{x^2-8}{x(x+7)^2}}$

D. $\ln\sqrt[5]{\dfrac{2(x+7)}{x(x^2-8)}}$

E. $\ln\sqrt[5]{\dfrac{(x+7)^2}{x(x^2-8)}}$

3. Given that $f(x) = 3 - x^2 - x$, find and simplify the difference quotient

$$\frac{f(2+h) - f(2)}{h}$$

A. $1 + h$

B. $\dfrac{3 - h^2 - h}{h}$

C. $-1 - h$

D. $5 + h$

E. $-5 - h$

4. If $f(x) = \sqrt{1 - 2x^3}$, find $f^{-1}(x)$

A. $(2 - 2x^2)^{\frac{1}{3}}$

B. $\left(\dfrac{1 + x^2}{2}\right)^{\frac{1}{3}}$

C. $\left(\dfrac{1 - x^2}{2}\right)^{\frac{1}{3}}$

D. $\left(\dfrac{1 - x^2}{2}\right)^{3}$

E. $\left(\dfrac{1 + x^2}{2}\right)^{3}$

5. Calculate the limit

$$\lim_{x\to -1^+} \frac{x-4}{x^3(x+1)}$$

A. ∞

B. -2

C. -1

D. 1

E. $-\infty$

6. The function

$$G(x) = \begin{cases} 5x-13 & x<2 \\ -5 & x=2 \\ x-5 & x>2 \end{cases}$$

is NOT continuous at $x=2$ because

A. $G(2)$ is not defined

B. $\lim_{x\to 2} G(x)$ does not exist

C. $\lim_{x\to 2} \neq G(2)$

D. $G(2) \neq -5$

E. There is a vertical asymptote at $x=2$

7. At what point(s) (x, y) on the graph of $f(x) = \dfrac{x}{x-2}$ does the tangent line to $f(x)$ have a slope of $-\dfrac{1}{2}$?

A. $(0, 0)$ only

B. $(-\frac{1}{2}, \frac{1}{5})$ only

C. $(0, 0)$ and $(4, 2)$

D. $(0, 0)$ and $(-4, \frac{2}{3})$

E. There are no such points

8. Suppose that $g(x) = (f(x))^2 - f(x^2)$. Also suppose that $f(1) = 2$ and $f'(1) = 3$. Find $g'(1)$.

A. 0

B. 2

C. 4

D. 6

E. 8

9. The slope of the line tangent to the curve described by the implicit function $y^3x+y^2x^2=6$ at $(2,1)$ is

A. $-\frac{3}{2}$
B. -1
C. $-\frac{3}{14}$
D. 0
E. $-\frac{5}{14}$

10. If $f(x)=(1+3x)^{\sin 2x}$, find $f'(\frac{\pi}{4})$

A. 3
B. $\frac{3}{1+\frac{3\pi}{4}}$
C. 2
D. $\left(1+\frac{3\pi}{4}\right)^{1-\frac{\pi}{4}}$
E. $\left(1+\frac{3\pi}{4}\right)\ln\left(1+\frac{3\pi}{4}\right)$

11. If $f(3) = 3$ and $f'(x) \geq 2$ for $3 \leq x \leq 6$, use the Mean Value Theorem to determine the smallest possible value of $f(6)$.

A. 2

B. 3

C. 6

D. 9

E. 18

12. Compute

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 4x + 1} - x \right)$$

A. -2

B. 0

C. 2

D. ∞

E. 4

13. Use a linear approximation to estimate the value of $\sqrt{36.3}$

A. 6.030
B. 6.025
C. 6.020
D. 6.015
E. 6.010

14. The derivative of the function $f(x)$ is given by

$$f'(x) = (x-1)(x)(x+1)^2(x+2)$$

The function $f(x)$ has a local minimum at

A. $x = -2$ and $x = 1$ only
B. $x = -2$ only
C. $x = -2$ and $x = 0$ only
D. $x = -1$ and $x = 0$ only
E. $x = -1$ and $x = 1$ only

15. A box with a square base and an open top must have a volume of 4 m^3. Find the height of the box that has the smallest possible surface area.

A. $\frac{3}{2}$

B. 4

C. $\frac{1}{2}$

D. 1

E. $\frac{1}{4}$

16. Use a Riemann sum with two rectangles and the right end points as the sample points to estimate the area under the curve $f(x) = 3^x$ from $x = 0$ to $x = 2$.

A. 4

B. 6

C. 8

D. 10

E. 12

17. The expression

$$\frac{1}{5}\left(\sqrt{\frac{1}{5}}+\sqrt{\frac{2}{5}}+\sqrt{\frac{3}{5}}+\sqrt{\frac{4}{5}}+\sqrt{\frac{5}{5}}\right)$$

is a Riemann sum approximation for

A. $\displaystyle\int_0^1 \sqrt{\frac{x}{5}}\,dx$

B. $\displaystyle\int_0^1 \sqrt{x}\,dx$

C. $\displaystyle\frac{1}{5}\int_0^1 \sqrt{\frac{x}{5}}\,dx$

D. $\displaystyle\frac{1}{5}\int_0^1 \sqrt{x}\,dx$

E. $\displaystyle\frac{1}{5}\int_0^5 \sqrt{x}\,dx$

18. Suppose that $g(x)$ is continuous and that $\displaystyle\int_4^7 g(x)\,dx = 10$ and $\displaystyle\int_4^{10} g(x)\,dx = 13$. Find $\displaystyle\int_7^{10} g(x)\,dx$.

A. 23

B. -23

C. 3

D. -3

E. Cannot be determined without knowing $g(x)$

19. Evaluate

$$\int_1^e \frac{(\ln x)^3}{x}\, dx$$

A. $\frac{1}{3}$

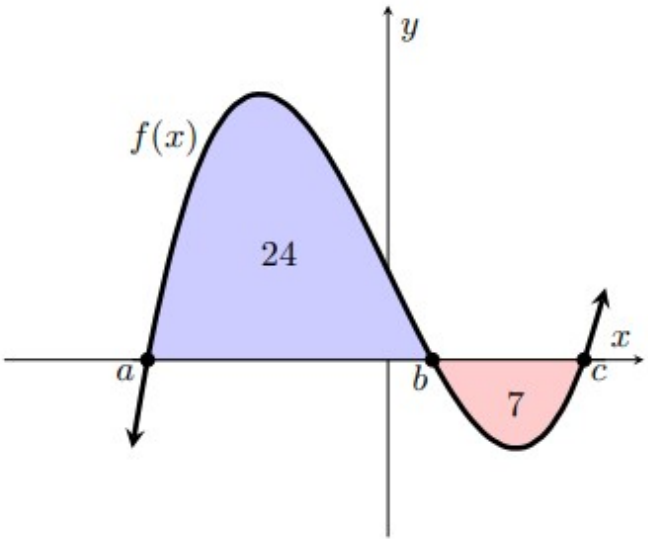
B. $\frac{e}{4}$

C. e

D. e^2

E. $\frac{1}{4}$

20. Below is the graph of $f(x)$. Evaluate $\int_a^c f(x)\, dx$



A. 31

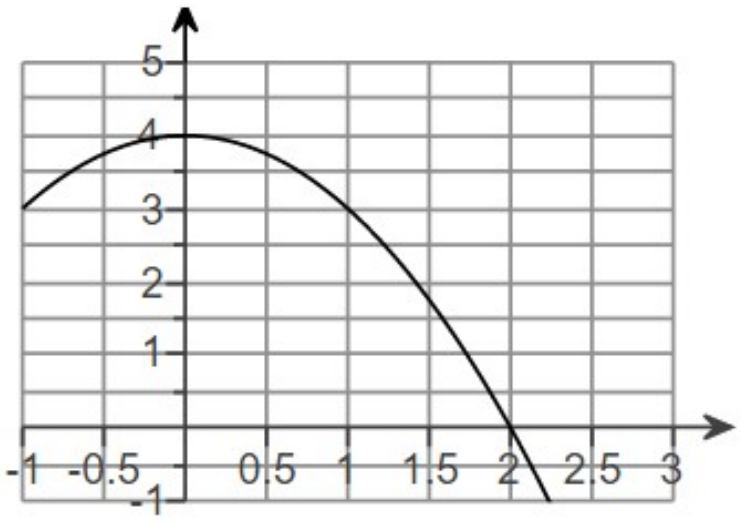
B. 24

C. -7

D. 15

E. 17

21. Below is the graph of $f(t)$. Let $g(x) = \int_0^x f(t)\,dt$. Determine the values of $g(0)$, $g'(0)$, and $g'(2)$.



A. $g(0) = 1$, $g'(0) = 0$, $g'(2) = 1$
B. $g(0) = 0$, $g'(0) = 0$, $g'(2) = 1$
C. $g(0) = 0$, $g'(0) = 0$, $g'(2) = 0$
D. $g(0) = 0$, $g'(0) = 4$, $g'(2) = 0$
E. $g(0) = 1$, $g'(0) = 4$, $g'(2) = 1$

22. Evaluate

$$\int_{-1}^{1} 3(|x| + x^2)\,dx$$

A. $\frac{3}{2}$
B. $\frac{5}{2}$
C. -3
D. -5
E. 5

23. Find the function F that satisfies the following equations and conditions

$$F'''(x) = 10x, F''(0) = 0, F'(0) = 4, F(0) = 5$$

A. $\frac{5}{4}x^3 + 4x + 5$
B. $\frac{5}{12}x^4 + 4x + 5$
C. $\frac{5}{12}x^4 + 5x + 4$
D. $\frac{5}{2}x^3$
E. $\frac{5}{4}x^3 + 5x + 4$

24. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of

$$\int_0^{\sin t} \frac{1}{25 - u^2}\, du$$

A. $\dfrac{1}{25 - \sin^2 t}$
B. $\dfrac{\cos t}{25 - \sin^2 t}$
C. $\dfrac{1}{\cos t(25 - \sin^2 t)}$
D. $\dfrac{-\cos t}{25 - \sin^2 t}$
E. $\dfrac{\sin t}{25 - \cos^2 t}$

25. A spacecraft uses a radioactive substance called unobtainium to generate power. A year ago there were 9 grams of unobtainium. Today there are 3 grams. What is the half-life of unobtainium?

A. $\dfrac{\ln 3}{\ln 9}$

B. $\dfrac{\ln 2}{\ln 3}$

C. $\dfrac{\ln 3}{\ln 4}$

D. $\ln 3$

E. $\ln 9$