MA 16100
FINAL EXAM Form A
Spring 2018

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

1. You must use a #2 pencil on the mark-sense sheet (answer sheet).

2. On the scantron, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces. BE SURE TO INCLUDE THE TWO LEADING ZEROS.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the scantron.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets.

9. Turn in both the scantron and the exam booklet when you are finished.

10. You cannot turn in your exam during the first 20 min or the last 10 min of the exam period.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should be put away and should not be visible at all. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Name:

[8 pt] If $g(t) = \dfrac{t^2}{t + \frac{3}{t}}$ then $g'(1) =$

1.A○ $\frac{1}{2}$

B○ $-\frac{1}{2}$

C○ 2

D○ $\frac{5}{8}$

E○ $\frac{3}{8}$

[8 pt] Use a linear approximation to estimate $e^{-0.01}$.

2.A○ 0.9

B○ 0.99

C○ 0.999

D○ 1.01

E○ 1.001

[8 pt] Let $f(x)$ be a polynomial with $f(2) = 1$. Assume that $f'(x) \geq 3$ for every x in $[2, 4]$.

What is the smallest possible value of $f(4)$?

Hint: Apply the Mean Value Theorem.

3.A○ 1

B○ 3

C○ 4

D○ 6

E○ 7

[8 pt] Evaluate the following limit: $\lim_{x \to \infty} \left(1 + \frac{1}{2x}\right)^{3x}$

4.A○ 1

B○ $e^{3/2}$

C○ e^3

D○ $e^{2/3}$

E○ e^5

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8 pt A 5 foot person walks at 40 feet per minute on a level path toward a vertical wall. A light on the ground directly behind the person casts a shadow of the person on the wall. The light is 100 feet from the wall. How fast is the person's shadow on the wall changing in length (in feet per minute) when the person is 25 feet from the wall?

5.A○ $-\frac{32}{9}$

B○ -32

C○ $-\frac{8}{9}$

D○ $-\frac{125}{14}$

E○ $-\frac{125}{28}$

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8 pt Suppose $f(x) = e^{x^2}$. Find $f''(x)$.

6.A○ $4x^2e^{x^2}$

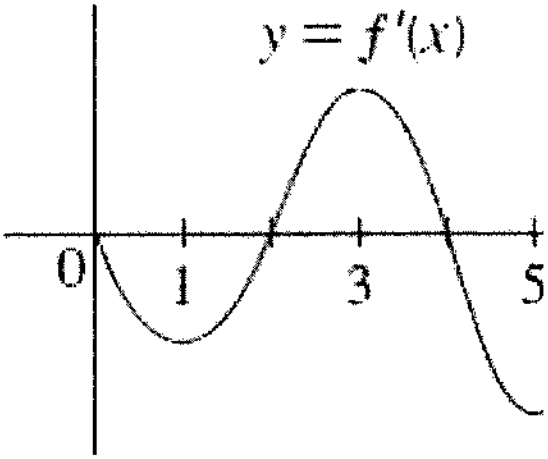
B○ $8x^2e^{x^2}$

C○ $2xe^{2x}$

D○ $(2+2x)e^{x^2}$

E○ $\left(2+4x^2\right)e^{x^2}$

[8 pt] The graph of $y = f'(x)$, the **derivative** of f, is shown below.



Which of the following statements about f are true?

I. The graph of f is concave up on the interval $(2, 4)$.

II. $f(x)$ has a local minimum at $x = 2$.

III. $(1, f(1))$ is an inflection point for f.

7.A○ None of these statements are true.

B○ I and III

C○ II and III

D○ I and II

E○ I, II, and III

[8 pt] For $-1 \leq x \leq 1$, $\tan\left(\sin^{-1} x\right)$ equals

8.A○ $\sqrt{1-x^2}$

B○ $\dfrac{\sqrt{1-x^2}}{x}$

C○ $\dfrac{x}{\sqrt{1-x^2}}$

D○ $x\sqrt{1-x^2}$

E○ $\dfrac{1}{\cos x}$

[8 pt] $\displaystyle\int_0^1 x^2\left(x^3+1\right)^{17}\,dx =$

9.A○ $\dfrac{2^{18}}{18}$

B○ $\dfrac{2^{18}}{54}$

C○ $\dfrac{2^{18}-1}{18}$

D○ $\dfrac{2^{18}-1}{54}$

E○ $\dfrac{2^{18}-1}{3}$

[8 pt] Evaluate $\frac{dy}{dx}$ if $y = \cos(\tan(3x))$.

10.A○ $-\sin(\tan(3x))\sec^2(3x)$

B○ $-3\sin(\tan(3x))\sec^2(3x)$

C○ $-3\sin\left(\sec^2(3x)\right)$

D○ $-\sin\left(\sec^2(3x)\right)$

E○ $-3\sin(\tan(3x))\sec(3x)\tan(3x)$

[8 pt] $\lim_{x\to\infty} \frac{2x^2-2x+1}{3-4x^2}$ equals

11.A○ 0

B○ 1

C○ $\frac{1}{2}$

D○ $-\frac{1}{4}$

E○ $-\frac{1}{2}$

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8 pt The graph $F(x) = 4 - 2^{x/3}$ is obtained from the graph $G(x) = 2^x$ by the following steps:

12.A○ Compress horizontally by a factor of 3, then reflect about the x axis, then shift up by 4 units

B○ Strech horizontally by a factor of 3, then reflect about the x axis, then shift up by 4 units

C○ Compress horizontally by a factor of 3, then reflect about the y axis, then shift up by 4 units

D○ Strech horizontally by a factor of 3, then reflect about the y axis, then shift down by 4 units

E○ Strech horizontally by a factor of 3, then reflect about the y axis, then shift to the right by 4 units

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8 pt Find the slope of the tangent line to the curve $\ln(xy) = x^2 - y^2$ at $(x, y) = (1, 1)$.

13.A○ $\frac{1}{3}$

B○ 0

C○ -1

D○ $\frac{2}{3}$

E○ 1

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8 pt $\int_0^4 \frac{1}{\sqrt{1+2x}}\,dx =$

14.A○ $\sqrt{3}-1$

B○ 2

C○ $2\sqrt{3}-1$

D○ 4

E○ $2\sqrt{3}+1$

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8 pts

Given $g(x) = \int_0^{x^2} \frac{u}{u^2+2}\,du$ the value of $g'(2)$ is

15.A○ $\frac{8}{9}$

B○ $\frac{4}{9}$

C○ $\frac{3}{2}$

D○ $\frac{3}{4}$

E○ 3

[8 pt] A particle is moving with acceleration function $a(t) = t - 2$, $v(0) = 3$ meters/second, and $s(0) = 1$ meter. Find the position, in meters, of the particle after 2 seconds.

16.A○ 1/2

B○ 5/6

C○ 19/6

D○ 13/3

E○ 19/3

[8 pt] If 40% of a certain radioactive substance decays in 50 days, what is the half-life of the substance?

17.A○ $50\frac{\ln 0.5}{\ln 0.4}$

B○ $50\frac{\ln 0.5}{\ln 0.6}$

C○ $50\frac{\ln 0.6}{\ln 0.5}$

D○ $50\frac{\ln 0.4}{\ln 0.5}$

E○ $50\frac{\ln 0.4}{\ln 0.6}$

[8 pt] Put the numbers $\log_9(\frac{1}{3})$, $\sinh(0)$, and $\cos(\tan^{-1}(-1))$ in the correct order.

18.A○ $\log_9(\frac{1}{3}) \leq \cos(\tan^{-1}(-1)) \leq \sinh(0)$

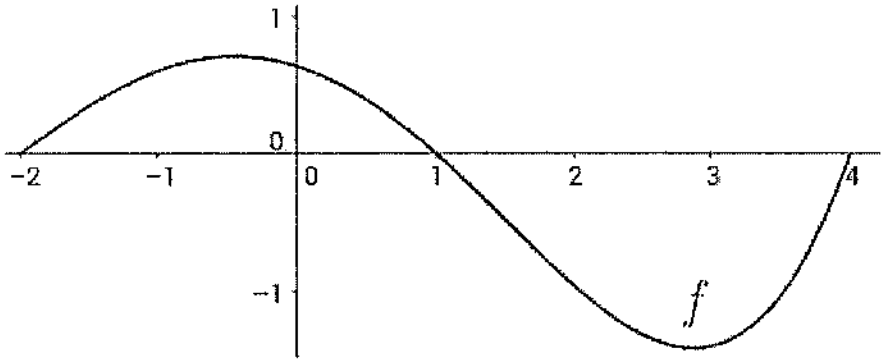
B○ $\cos(\tan^{-1}(-1)) \leq \log_9(\frac{1}{3}) \leq \sinh(0)$

C○ $\sinh(0) \leq \cos(\tan^{-1}(-1)) \leq \log_9(\frac{1}{3})$

D○ $\sinh(0) \leq \log_9(\frac{1}{3}) \leq \cos(\tan^{-1}(-1))$

E○ $\log_9(\frac{1}{3}) \leq \sinh(0) \leq \cos(\tan^{-1}(-1))$

[8 pt] Suppose $A(x) = \int_0^x f(t)\,dt$ where the graph of f is pictured below.



At what x value does $A(x)$ attain its maximum on the interval $-2 \leq x \leq 4$?

19.A○ $x = -2$

B○ $x = 0$

C○ $x = 1$

D○ $x = 3$

E○ $x = 4$

8 pt In this picture of a circular sector the area $\frac{1}{2}r^2\theta$ and the length of the circular arc is $r\theta$, where r is the radius and θ is the opening angle. What is the maximum area among circular sectors whose perimeter is 6?

20.A○ 3.5

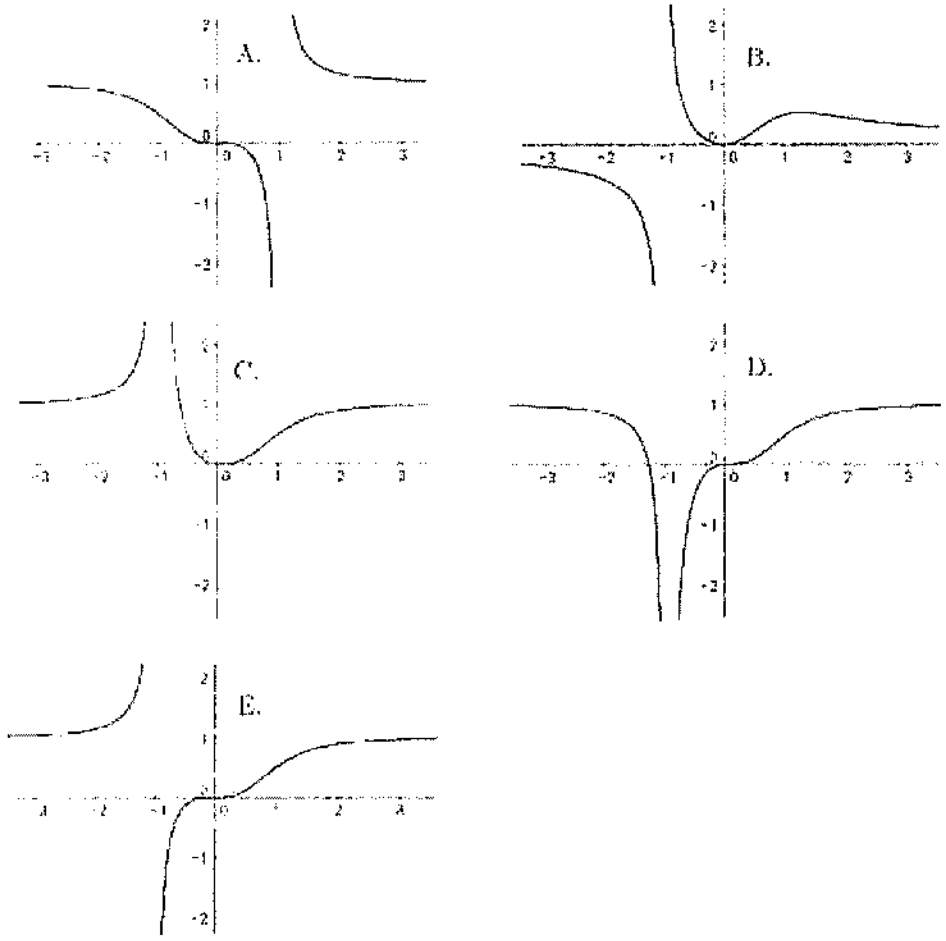
B○ 4

C○ 1.75

D○ 3

E○ 2.25

8 pt Which of the following is the graph of $y = \dfrac{x^3}{x^3+1}$?



21.A○ A

B○ B

C○ C

D○ D

E○ E

8 pt Let f be a function whose derivative f' is given by $f'(x) = (x-2)^2 x^4 (x+2)^5$. Then f has a

22.A○ local minimum at $x = 0$

B○ local minimum at $x = -2$

C○ local maximum at $x = 0$

D○ local minimum at $x = 2$

E○ local maximum at $x = 2$

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8 pt Assume a is a positive number. If $\int_1^{\sqrt{a}} \frac{1}{x}\,dx = 3$, then $\int_1^{a} \frac{1}{x}\,dx$ equals

23.A○ 9

B○ 6

C○ $\sqrt{3}$

D○ 12

E○ $\ln 9$

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8 pt The velocity of a particle moving on a line at time t is $v(t) = 3t^{1/2} + 5t^{3/2}$ meters/second. What distance did it travel, in meters, from $t = 0$ to $t = 4$?

24.A○ 40

B○ 32

C○ 80

D○ 64

E○ 184

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8 pt Find the domain of the function $f(x) = \sqrt{\ln(x-1)}$.

25.A○ $(1, \infty)$

B○ $(1, 2]$

C○ $[2, \infty)$

D○ $[1 + e, \infty)$

E○ $(2, 1 + e]$