MA 16100
FINAL EXAM Green
May 5, 2016

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. Be sure the paper you are looking at right now is GREEN!

3. Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes):

0 1

4. On the mark-sense sheet, fill in your TA's name and the course number.

5. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

6. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

7. Sign the mark–sense sheet.

8. Fill in your name, etc. on this paper (above).

9. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets.

10. Turn in both the mark–sense sheets and the question sheets when you are finished.

11. If you finish the exam before 9:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 9:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

12. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. The function $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ has

A. No vertical asymptotes and no horizontal asymptotes.

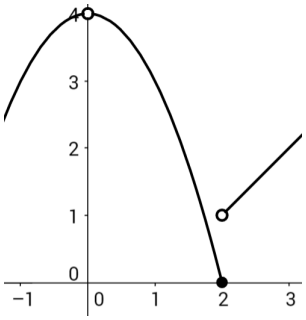
B. No vertical asymptotes and one horizontal asymptote.

C. One vertical asymptote and one horizontal asymptote.

D. Two vertical asymptotes and no horizontal asymptotes.

E. Two vertical asymptotes and one horizontal asymptote.

2. The graph of $y = f(x)$ is pictured below.



A. $\lim\limits_{x \to 0} f(x) = 4$ and $\lim\limits_{x \to 2^+} f(x)$ does not exist.

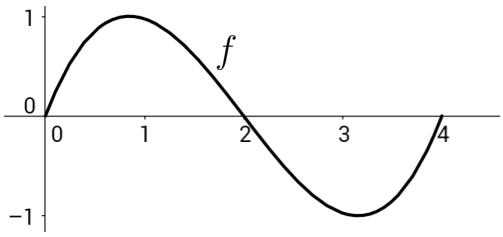
B. $\lim\limits_{x \to 0} f(x) = 4$ and $\lim\limits_{x \to 2^+} f(x) = 0$.

C. $\lim\limits_{x \to 0} f(x)$ does not exist and $\lim\limits_{x \to 2^+} f(x) = 0$.

D. $\lim\limits_{x \to 0} f(x)$ does not exist and $\lim\limits_{x \to 2^+} f(x) = 1$.

E. $\lim\limits_{x \to 0} f(x) = 4$ and $\lim\limits_{x \to 2^+} f(x) = 1$.

3. The graph of $y = f(x)$ is pictured below.



Suppose $g(x) = \dfrac{1}{\sqrt{f(x)}}$. What is the domain of g?

A. $[0, 4]$

B. $(0, 1]$

C. $[-1, 1]$

D. $(0, 2)$

E. $(0, 2) \cup (2, 4)$

4. Find the value of a so that f is continuous for all real numbers.

$$f(x) = \begin{cases} a^{-x} & x \leq 1 \\ \dfrac{\sqrt{x^2+3}-2}{x^2-1} & x > 1 \end{cases}$$

A. 8

B. 4

C. 2

D. 1

E. Such an a does not exist.

5. Find the following limit:

$$\lim_{h\to 0} \frac{\sin(\frac{7\pi}{6}+h)-\sin(\frac{7\pi}{6})}{h}$$

A. $\dfrac{7\pi}{6}$

B. $\dfrac{1}{2}$

C. $-\dfrac{\sqrt{3}}{2}$

D. $-\dfrac{1}{2}$

E. $-\dfrac{7\sqrt{3}}{12}$

6. Suppose $f(1)=2$, $f'(1)=\dfrac{3}{4}$, and $y=2^{f(x)}$. Find $\dfrac{dy}{dx}$ at $x=1$.

A. 3

B. $2^{3/2}$

C. $\ln 6$

D. $\ln 8$

E. $2^{3/4}$

7. Find the slope of the tangent line to the curve

$$\ln(xy) = x^2 - y^2$$

at $(x, y) = (1, 1)$.

A. $\frac{1}{3}$

B. 0

C. -1

D. $\frac{2}{3}$

E. 1

8. Find the equation of the tangent line to

$$y = \tan^{-1}(2 - x^2)$$

at $x = 1$.

A. $y = -x + \frac{\pi}{4} + 1$

B. $y = x + \frac{\pi}{4} - 1$

C. $y = \frac{x}{2} + \frac{\pi}{4} - \frac{1}{2}$

D. $y = -x + 1$

E. $y = -\frac{x}{5} + \frac{6}{5}$

9. If $f(x) = x^{\ln x}$, find $f'(e)$. *Hint: "logarithmic differentiation."*

A. e

B. 2

C. 0

D. 1

E. $1/e$

10. A particle moves with position function

$$s = t^4 - 5t^3 + 6t^2 + 8t.$$

During what time interval is the acceleration negative?

A. $(0, 6)$

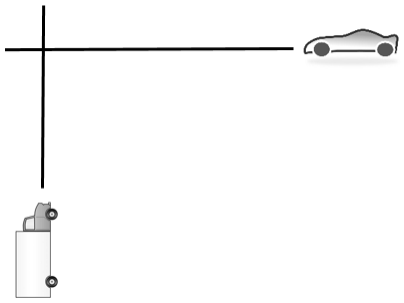
B. $(2, 6)$

C. $(0, 2)$

D. $(\frac{1}{2}, 2)$

E. $(\frac{1}{2}, 6)$

11. A car is traveling west at 30 mi/hr and a truck is traveling north at 20 mi/hr. Both are headed for the intersection of the two roads. At what rate are the vehicles approaching each other when the car is 2 mi from the intersection and the truck is 1 mi from the intersection?



A. $10\sqrt{13}$ mi/hr

B. $16\sqrt{5}$ mi/hr

C. $20\sqrt{10}$ mi/hr

D. 25 mi/hr

E. $20\sqrt{3}$ mi/hr

12. Find the critical numbers of $f(x) = x^{2/3}(6-x)^{1/3}$.

A. $x = 0$ and $x = 6$

B. $x = 12$

C. $x = 0$ and $x = 12$ and $x = 6$

D. $x = 0$ and $x = 4$ and $x = 6$

E. $x = 2$ and $x = 4$

13. Find the limit.

$$\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$$

A. 0

B. ∞

C. $-\infty$

D. 1

E. -1

14. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(4, 0)$.
Hint: The distance between two points (a, b) *and* (x, y) *is* $\sqrt{(x-a)^2 + (y-b)^2}$.

A. $(4, 2)$

B. $(0, 0)$

C. $(1, 1)$

D. $(2, \sqrt{2})$

E. $\left(3.5, \sqrt{3.5}\right)$

15. Find an x-value at which $f(x) = x + \cot(x/2)$ has a local minimum.

A. $x = \pi/4$

B. $x = \pi/2$

C. $x = 3\pi/4$

D. $x = \pi$

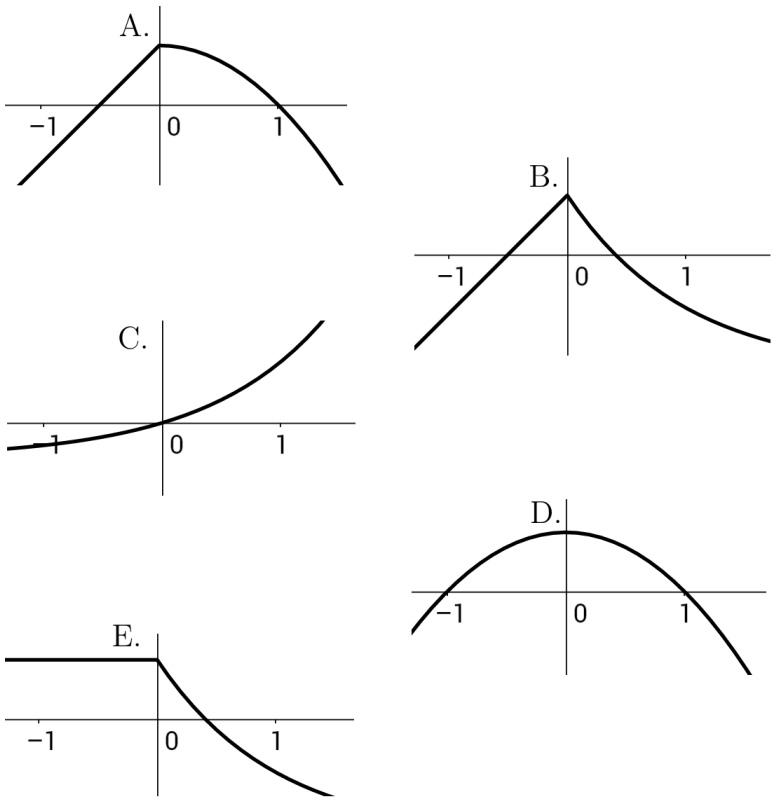
E. $x = 5\pi/4$

16. Which of these graphs satisfies all three of the following conditions?

$f'(x) > 0$ for $x < 0$.

$f'(x) < 0$ for $x > 0$.

$f''(1) > 0$.



17. Estimate $\int_0^{\pi} \sin x \, dx$ using a right Riemann sum with $n = 4$ rectangles.

A. $\frac{\pi}{4} + \frac{\pi\sqrt{2}}{4}$

B. $1 + \sqrt{2}$

C. $\frac{7}{4}$

D. $\frac{1+\sqrt{2}}{4}$

E. $\frac{5\pi}{2}$

18. Find an antiderivative of the function $f(x) = \sec x \tan x$.

A. $\sec^3 x \tan x$

B. $\sec x$

C. $\ln|\sec x|$

D. $\tan x$

E. $\frac{1}{2}\sec^2 x$

19. Suppose that $\int_1^8 f(x)\,dx = 4$, $\int_5^8 f(x)\,dx = 7$, and $\int_5^1 g(x)\,dx = 3$. Find

$$\int_1^5 (f(x) - 2g(x))\,dx.$$

A. -9

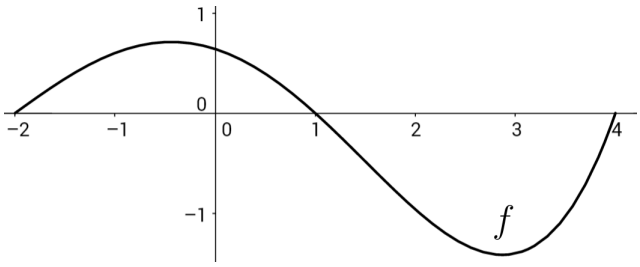
B. 0

C. 1

D. 3

E. 5

20. Suppose $A(x) = \int_0^x f(t)\,dt$ where the graph of f is pictured below.



At what x value does $A(x)$ attain its maximum on the interval $-2 \le x \le 4$?

A. $x = -2$

B. $x = 0$

C. $x = 1$

D. $x = 3$

E. $x = 4$

21. Suppose

$$f(x) = \int_1^{\ln x} t^3 e^t \, dt.$$

Find $f'(x)$.

A. $x(\ln x)^3$
B. $x^3 e^x$
C. $x^2 e^x$
D. $e^x(\ln x)^3$
E. $(\ln x)^3$

22. $\displaystyle\int_1^2 \frac{x^2 - 1}{x} \, dx$

A. $\dfrac{9}{2}$
B. $\dfrac{3}{2}$
C. $\dfrac{9}{4}$
D. $\dfrac{3}{2} - \ln 2$
E. $5 - \ln 2$

23. $\displaystyle\int 2x^5 e^{x^6}\, dx$

A. $2e^{x^6} + C$

B. $\dfrac{1}{3}e^{x^6} + C$

C. $\dfrac{1}{3}x^6 + 6x^5 e^{x^6} + C$

D. $10x^4 e^{x^6} + 12x^{10} e^{x^6} + C$

E. $12e^{x^6} + C$

24. $\displaystyle\int_1^e \frac{(\ln x)^3}{x}\, dx$

A. $\dfrac{1}{4}$

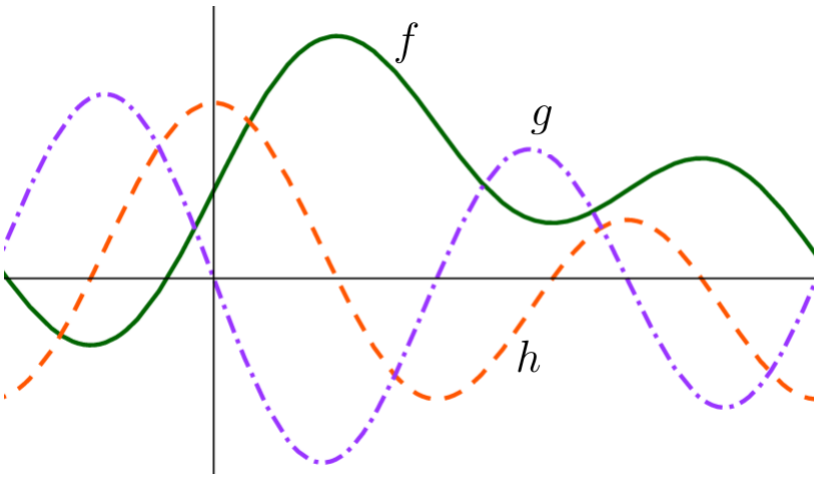
B. $\dfrac{e^4 - 1}{4}$

C. 1

D. $\dfrac{1}{e}$

E. $e^3 - 1$

25. The graph of a function, its derivative, and one of its antiderivatives is pictured below.



A. f is an antiderivative of g and h is the derivative of g.

B. h is an antiderivative of g and f is the derivative of g.

C. g is an antiderivative of f and h is the derivative of f.

D. h is an antiderivative of f and g is the derivative of f.

E. f is an antiderivative of h and g is the derivative of h.