MA 161/161E Final Exam Spring 2007

Name:

10-digit PUID:

Lecturer:

Recitation Instructor:

Recitation Time:

Instructions:

1. This package contains 25 problems worth 8 points each.

2. Please supply all the information requested. On the scantron sheet, print your name, your division-section number, and 10 digit PUID number in addition to filling in the corresponding circles.

3. Work only in the space provided, or on the backside of the pages. Circle your choice for each problem in this booklet, and mark your answer on the scantron sheet.

4. No books, notes, calculator, or any electronic devices may be used on this exam.

1 E 2 D 3 E 4 C 5 A

6 C 7 A 8 B 9 A 10 C

11 C 12 D 13 A 14 A 15 B

16 D 17 A 18 D 19 C 20 D

21 E 22 E 23 B 24 B 25 E

1. Consider the set of points (x, y) satisfying the equation

$$x^2 + y^2 - 4x - 4y + 7 = 0.$$

Which of the following statements are true?

I. It meets the x-axis.

II. It meets the y-axis.

III It includes the point $(0, 0)$.

A. Only I and II

B. Only II and III.

C. Only I and III.

D. All of them.

E. None of them.

2. f and g are functions defined on the entire real line and f is increasing while g is decreasing. Which of the following statements are true?

I. $g \circ f$ is decreasing.

II. $g \circ f$ is increasing.

III. $f \circ g$ is decreasing.

IV. $f \circ g$ is increasing.

A. Only I.

B. Only II.

C. Only II and IV.

D. Only I and III.

E. Only III and IV.

3. What is the domain of the inverse function f^{-1} if $f(x) = \dfrac{e^x - 1}{2}$?

A. $(-\infty, \infty)$

B. $(0, \infty)$

C. $(2, \infty)$

D. $(1/2, \infty)$

E. $(-1/2, \infty)$

4. Which of the following limits exist?

I. $\lim_{x \to 0+} (1 + e^{1/x})^{-1}$.

II. $\lim_{x \to 0-} (1 + e^{1/x})^{-1}$.

III. $\lim_{x \to 0} (1 + e^{1/x})^{-1}$.

A. Only I.

B. Only II.

C. Only I and II.

D. Only II and III.

E. None of them.

5. What is

$$\lim_{x\to 0^+} \cos^{-1}\left(\frac{\sqrt{x}-1}{x-1}\right) \quad ?$$

(Here, $\cos^{-1}$ refers to the inverse function of cosine.)

A. 0

B. 1

C. $\pi/3$

D. $\pi/6$

E. It does not exist.

6. Let $f(x) = \frac{1}{3}x^3 + x^2 + 2x - 1$. Which of the following statements are true?

I. The intermediate value theorem guarantees that $f(x) = 0$ has a solution in the interval $[-2, 2]$.

II. The mean-value theorem guarantees that $f(x) = 0$ has at most one solution in $[-2, 2]$.

A. Only I.

B. Only II.

C. Both.

D. Neither.

E. None of the answers above is correct.

7. Which of the following functions are continuous?

   I. The distance from the launch pad of a rocket sent into space, as a function of time.

   II. The price of postage for sending a letter from West Lafayette to New York City as a function of the weight.

   III. The taxi-fare in West Lafayette as a function of the distance traveled.

   A. Only I.

   B. Only II.

   C. Only III.

   D. Only I and II.

   E. Only II and III.

8. Which of the following statements about the function $\sqrt{x+1} - \sqrt{x}$ are true?

   I. It is defined on the interval $(-1, \infty)$.

   II. It has a horizontal asymptote.

   III. It has a vertical asymptote.

   A. Only I.

   B. Only II.

   C. Only III.

   D. Only I and III.

   E. All of them.

9. Let $f(x) = \sin(2x)$. What is $f^{(66)}(0)$?

A. 0

B. 1

C. -1

D. π

E. None of the above.

10. What is the derivative of x^{e^x} at $x = 1$?

A. 0

B. 1

C. e

D. $\ln(e)$

E. None of the above.

11. Let $f(x) = \sqrt{1-x^2}$. What is the domain of its derivative $f'(x)$?

A. $[1, \infty)$

B. $(1, \infty)$

C. $(-1, 1)$

D. $[-1, 1]$

E. $(-\infty, 1]$.

12. Let L be the tangent line to

$$\sqrt{x} + \sqrt{y} = 1$$

at the point $(1/4, 1/4)$. Which of the following lines is parallel to L?

A. $y = (1/4)x + 2$

B. $x - y = 1$

C. $y = 0$

D. $\pi y + \pi x = 13$

E. None of the above.

13. What is the derivative of

$$y = \frac{\sin^2(x)\tan^4(x)}{x^2}$$

at $x = \pi/4$?

A. $80/\pi^2 - 64/\pi^3$

B. $\pi^2/6$

C. 1

D. $100\pi - 6/\pi^2$

E. None of the above.

14. If the area of a rectangle grows at 20 in^2/min and its base grows at 2 in/min, at what rate is the height growing when the base is 8 in and the height is 6 in?

A. 1 in/min

B. 2 in/min

C. 6 in/min

D. 20/7 in/min

E. 10 in/min

15. How many local minima does

$$\frac{x}{2} - \sin(x)$$

have on $(0, 2\pi)$?

A. 0

B. 1

C. 2

D. 3

E. 4

16. The function $x^4 + 2x^3 - 4$ is concave up on

A. $(-\infty, 0)$

B. $(-\infty, 1)$

C. $(-1, 1)$

D. $(0, \infty)$

E. $(-1, 0)$

17. $\lim_{x\to\infty} e^x \ln(1+e^{-x}) =$

A. 1

B. 0

C. -1

D. ∞

E. $-\infty$

18. The sum of three positive numbers is 12. If one of them is three times another, what is the maximum possible value for the product of these three numbers?

A. 24

B. 56

C. 42

D. 48

E. 62

19. To approximately solve the equation $x^3 - 2x + 6 = 0$ by Newton's method, we start with the initial guess $x_1 = 2$. Then $x_2 =$

A. -1

B. 0

C. 1

D. 3/2

E. 5/2

20. If $g'(x) = x + \frac{1}{x}$ and $g(1) = 1$, then $g(3) =$

A. 3

B. 3/2

C. $3 \ln 3/2$

D. $5 + \ln 3$

E. $2 + \ln 3$

21. If $\int_3^0 f(x)dx = -3$ and $\int_3^5 f(x)dx = 5$, then $\int_0^5 f(x)dx =$

A. 2

B. 0

C. 3

D. 5

E. 8

22. Suppose f is a continuous function on $(-\infty, \infty)$ and $\int_1^x f(t)dt = g(x)$. Which is true?

I. $g'(x) = f(x)$.

II. If $f(x) > 0$ for all x, then $g(x)$ is an increasing function.

III. If $h(x) = g(x^2)$, then $h'(1) = 2f(1)$.

A. Only I.

B. Only II.

C. Only III.

D. Only I and II.

E. All are true.

23. $\int_0^3 (1-x^2)dx=$

A. -9

B. -6

C. -8

D. 4

E. 9

24. Which substitution should be used to evaluate $\int \sin(x)e^{\cos(x)}dx$?

A. $u = \sin(x)$

B. $u = \cos(x)$

C. $u = e^{\cos(x)}$

D. $u = e^{-\cos(x)}$

E. $u = -e^{-\cos(x)}$

25. t minutes after being filled, a bucket leaks water at the rate of

$$\frac{1}{(t+1)^2}$$

fluid ounces per minute. How much water (in fluid ounces) will be lost in the first 9 minutes?

A. 4

B. 1/10

C. 9

D. 10

E. 9/10