MA 16100
FINAL EXAM
12/13/2022
TEST/QUIZ NUMBER:
00

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough space for your name, fill in as much as you can.
2. Section number with a leading zero, e.g. **0302**. (If you don't know your section number, ask your TA.)
3. Test/Quiz number: **00**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros**
5. Blacken in your choice of the correct answer on the scantron answer sheet for questions 1–12.

There are **25** questions, each worth 4 points, for a total of 100 points. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 9:50am, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20am. If you don't finish before 9:50am, you must remain seated until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam booklet until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, students must put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: ______________________________

1. For what values of x does the graph of $f(x) = \tan^{-1}\left(\frac{x}{4}\right) + \frac{2}{x}$ have a horizontal tangent line?

A. The graph does not have a horizontal tangent line.

B. $x = 2$ and $x = -2$

C. $x = 2$ and $x = 4$

D. $x = 0$

E. $x = 4$ and $x = -4$

2. The function

$$f(x) = \begin{cases} 1, & x < -1 \\ -x, & -1 \leq x \leq 1 \\ \sin(\pi x), & x > 1 \end{cases}$$

is not continuous at:

A. $x = -1$

B. $x = -1$ and $x = 1$

C. $x = 1$

D. $x = 0$ and $x = 1$

E. $f(x)$ is continuous everywhere.

3. If $g(x) = \int_0^{2x} e^{t^2}\, dt$, then $g'(x) =$

A. $2e^{2x^2}$

B. e^{2x^2}

C. e^{4x^2}

D. $2e^{x^2}$

E. $2e^{4x^2}$

4. Evaluate:

$$\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)$$

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $-\frac{\pi}{3}$

D. $-\frac{2\pi}{3}$

E. $\frac{4\pi}{3}$

5. Evaluate:

$$\lim_{x\to\infty} x \sin\left(\frac{2}{x}\right)$$

A. 0

B. 2

C. 1

D. The limit does not exist and is neither ∞ nor $-\infty$

E. ∞

6. Use linear approximation to estimate $e^{-0.01}$.

A. 1.01

B. 1.1

C. 1

D. 0.9

E. 0.99

7. Evaluate

$$\int_0^{\frac{1}{3}} \frac{dx}{1+9x^2}$$

A. $\dfrac{\pi}{12}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{\ln(2)}{18}$

D. $\dfrac{\pi}{4}$

E. $\ln(2)$

8. Find the average value of $f(x) = \dfrac{1}{x}$ on the interval $[1,5]$.

A. $\dfrac{\ln(5)}{4}$

B. $\dfrac{5}{4}$

C. $\dfrac{3}{5}$

D. $\ln(5)$

E. $\dfrac{\ln(4)}{4}$

9. If

$$f(x) = \frac{x^2 - \sin(x)}{\cos(x)}$$

then $f'(x) =$

A. $2x\sec(x) - 1$

B. $2x\sec(x) - \sec^2(x)$

C. $-2x\csc(x) + \tan(x)$

D. $2x\sec(x) + x^2\tan(x)\sec(x) - \sec^2(x)$

E. $2x^2\tan(x)\sec(x) - \sec(x)$

10. A storage crate is to be built in the shape of a box with a square base. It is to have volume 10 cubic feet. The material for the base costs \$4 per square foot, the material for the lid costs \$1 per square foot, and the material for the sides costs \$2 per square foot. If a is the width of the crate and h is the height, what are the dimensions of the crate the minimizes the cost?

A. $a = 1$ and $h = 10$

B. $a = \sqrt{2}$ and $h = 5$

C. $a = \sqrt{5}$ and $h = 2$

D. $a = 2$ and $h = \frac{5}{2}$

E. $a = \sqrt{10}$ and $h = 1$.

11. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 3 feet per second. If the height of the streetlight is 18 feet, what is the rate at which the length of the person's shadow is growing?

A. $\frac{3}{2}$ feet per second

B. $\frac{1}{3}$ feet per second

C. $\frac{1}{2}$ feet per second

D. $\frac{1}{6}$ feet per second

E. 2 feet per second

12. Let $f(x) = e^{\tan(3x)}$. Find $f'(0)$.

A. 3

B. 1

C. $f(x)$ is not differentiable at $x = 0$.

D. $3e$

E. ∞

13. Suppose $f(x) = \dfrac{2x^3 + 16x^2 + 30x}{x^3 + 5x^2}$. Which of the following statements are correct?

(i) $y = 2$ is a horizontal asymptote.

(ii) $x = -5$ is a vertical asymptote.

(iii) $\lim\limits_{x \to 0^-} f(x) = \infty$.

A. Only statement (i) is correct.

B. Only statements (i) and (iii) are correct.

C. All three statements are correct.

D. Only statements (ii) and (iii) are correct.

E. Only statements (i) and (ii) are correct.

14. The limit $\lim\limits_{h \to 0} \frac{\cos(h^2)-1}{h}$ represents $f'(a)$, the derivative of some function f at some number a. Find such an f and a

A. $f(x) = \cos(x^2),\ a = 0$

B. None of the above

C. $f(x) = \cos(x),\ a = 1$

D. $f(x) = \cos(x^2),\ a = 1$

E. $f(x) = \cos(x),\ a = 0$

15. Find the slope of the line tangent to the curve $y = x\tan^{-1}(x)$ at the point $(1, \frac{\pi}{4})$.

A. $\dfrac{1}{2}$

B. $\dfrac{\pi}{4} + \dfrac{1}{2}$

C. $\dfrac{\pi}{4}$

D. $\dfrac{3\pi}{4}$

E. $\dfrac{\pi}{4} + 1$

16. Suppose that

$$\lim_{x\to 0} f(x) = 0 \qquad \text{and} \qquad \lim_{x\to 0} g(x) = -\infty$$

What can be said about $\lim\limits_{x\to 0} f(x)g(x)$?

A. $\lim\limits_{x\to 0} f(x)g(x)$ is either 0 or $-\infty$

B. $\lim\limits_{x\to 0} f(x)g(x) = -\infty$

C. $\lim\limits_{x\to 0} f(x)g(x) = -1$

D. $\lim\limits_{x\to 0} f(x)g(x) = 0$

E. The limit may or may not exist. If it exists, it can be any number

17. Find the integral

$$\int_{e}^{e^9} \frac{dx}{x\sqrt{\ln(x)}}$$

A. 1

B. 6

C. 2

D. 4

E. 3

18. Consider a function $f(x)$ defined such that $\displaystyle\int_1^8 f(x)dx = 12$ and $\displaystyle\int_5^8 f(x)dx = 7$

What is $\displaystyle\int_5^1 2f(x)dx$?

A. -7

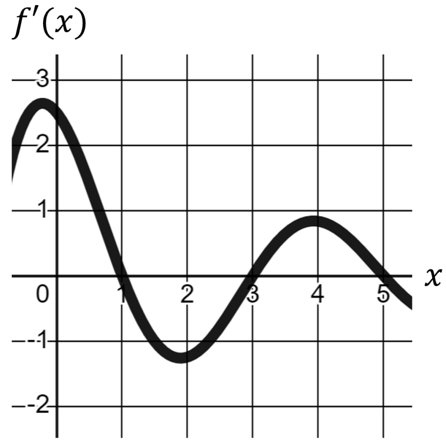
B. 5

C. -12

D. 14

E. -10

19. Consider a function $f(x)$. The graph of its **derivative** is plotted below.



Which of the following statements is true?

I. $f(x)$ is increasing on $(3, 5)$

II. $f(x)$ has a local minimum at $x = 1$

III. $f(x)$ is concave up on $(1, 3)$

A. II and III only

B. I, II, and III

C. I and III only

D. I only

E. I and II only

20. Consider the function $f(x) = e^{-x^2}$. On what interval is $f(x)$ both decreasing and concave upward?

A. $\left(\frac{1}{\sqrt{2}}, \infty\right)$

B. $(0, \infty)$

C. $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

D. $\left(0, \frac{1}{\sqrt{2}}\right)$

E. $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}}, \infty\right)$

21. What is the domain of the function $f(x) = \sqrt{16 - x^2} + \ln(1 - x)$?

A. $[-4, 4]$
B. $[0, 1)$
C. $[-4, 1)$
D. $(1, 4]$
E. $(0, 4]$

22. According to the Mean Value Theorem, the function $f(x) = 4x^2 \sin(\pi x)$ has slope $= 2$ on which of the following intervals?

I. $[-\frac{1}{2}, \frac{3}{2}]$
II. $[-\frac{1}{2}, 0]$
III. $[-\frac{1}{2}, \frac{1}{2}]$

A. I, II and III
B. I and II only
C. II only
D. II and III only
E. I and III only

23. At time $t = 0$ a toy rocket is launched. Its position after t seconds is $x(t) = 10\,(t - t^2)\,e^t$ meters above the ground. What is the speed in meters/sec of the rocket when it returns to the ground?

A. $\frac{20e}{3}$

B. $15e$

C. $10e$

D. $\frac{15e}{2}$

E. $5e$

24. Find the horizontal asymptotes of

$$f(x) = \frac{2e^x - e^{-x}}{e^x + 3e^{-x}}$$

A. $y = \frac{1}{3}$ and $y = -\frac{1}{3}$

B. $y = 2$ and $y = -2$

C. $y = 2$

D. $y = 2$ and $y = -\frac{1}{3}$

E. There are no horizontal asymptotes.

25. 30% of a radioactive substance decays in 5 hours. What is the half-life of the substance?

A. $\frac{1}{2}\left(\frac{\ln(0.3)}{\ln(5)}\right)$ hours

B. $5\left(\frac{\ln(0.3)}{\ln(\frac{1}{2})}\right)$ hours

C. $5\left(\frac{\ln(\frac{1}{2})}{\ln(0.3)}\right)$ hours

D. $30\left(\frac{\ln(\frac{1}{2})}{\ln(5)}\right)$ hours

E. $0.3\left(\frac{\ln(5)}{\ln(2)}\right)$ hours

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