MA 16100
FINAL EXAM Version 01
December 13, 2017

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

1. You must use a #2 pencil on the scantron

2. a. Write 01 in the TEST/QUIZ NUMBER boxes and darken the appropriate bubbles on your scantron.

   b. **The color of your scantron MUST match the color of the cover page of your exam**

3. On the scantron sheet, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the scantron sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 25 questions, each one is worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Also circle your answers on the exam itself. Do all your work on the question sheets.

9. Turn in both the scantron sheets and the question sheets when you are finished.

10. If you finish the exam before 12:20 pm, you may leave the room after turning in the scantron sheet and the exam booklet.
    If you don't finish before 12:20 pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

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EXAM POLICIES

1. Students may not open the exam until instructed to do so.

2. Students must obey the orders and requests by all proctors, TAs, and lecturers.

3. No student may leave in the first 20 min or in the last 10 min of the exam.

4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.

5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams. **ANY talking or writing during this time will result in an AUTOMATIC ZERO.**

6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. If the graph of the exponential function $f(x) = Ca^x$ passes through the points $(-1, 9)$ and $(1, 4)$, find the value of C.

    A. $\frac{1}{9}$

    B. 9

    C. 4

    D. $\frac{-5}{2}$

    E. 6

2. Suppose f is a function that is defined for all real numbers. Which of the following conditions assures that f has an inverse?

    A. f is always increasing

    B. f is periodic

    C. f is symmetric with respect to the y-axis

    D. f is concave downward

    E. f is continuous

3. At $x = 3$ the function $f(x) = \begin{cases} x^2 & x < 3 \\ 6x - 9 & x \geq 3 \end{cases}$ is

    A. Undefined

    B. Continuous but not differentiable

    C. Differentiable but not continuous

    D. Neither differentiable nor continuous

    E. Both continuous and differentiable

4. Evaluate the limit $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^6 - x}}{x^3 + 8}$

    A. ∞

    B. 2

    C. 1

    D. $\frac{1}{2}$

    E. 0

5. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

A. $\lim\limits_{h \to 0} \dfrac{e^h}{h}$

B. $\lim\limits_{h \to 0} \dfrac{e^h - e^e}{h}$

C. $\lim\limits_{h \to 0} \dfrac{e^h - 1}{h}$

D. $\lim\limits_{h \to 0} \dfrac{e^{x+h} - 1}{h}$

E. $\lim\limits_{h \to 0} \dfrac{e^{e+h} - e^e}{h}$

6. Find $f'(4)$ if $f(x) = \dfrac{2x^2 + 16}{\sqrt{x}}$.

A. 1

B. 2

C. 3

D. 4

E. 5

7. Find $f'(1)$ if $f(x) = \dfrac{8x}{\sqrt{x^2 + 3}}$.

    A. 1

    B. 2

    C. 3

    D. 4

    E. 5

8. Find $f'(1)$ if $f(x) = \ln\left[\dfrac{(3x - 1)^6}{(x + 1)^4(2x + 1)^3}\right]$.

    A. 1

    B. 2

    C. 3

    D. 4

    E. 5

9. Suppose that $f(x)$ and $g(x)$ are functions with $f(1) = 2$, $f'(1) = 9$, $g(1) = 2$ and $g'(1) = 4$. Let $h(x) = \dfrac{f(e^{2x})}{g(e^{3x})}$. Find $h'(0)$.

A. 1

B. 2

C. 3

D. 4

E. 5

10. A roast turkey is taken from an oven when its temperature has reached 175°F and is placed on a table in a room where the temperature is $T_s = 75°$F. After half an hour the temperature of the turkey is 125°F. What is the temperature, in °F, after 1 hour? Recall Newton's Law of Cooling states that $\dfrac{dT}{dt} = k(T - T_s)$.

A. $100 \ln 4$

B. 100

C. $100 \ln 2$

D. 75

E. 50

11. The angle of elevation of the Sun (θ, see figure below) is decreasing at 0.25 rad/hour. How fast is the shadow cast by a 400-ft building increasing when $\theta = \dfrac{\pi}{6}$?
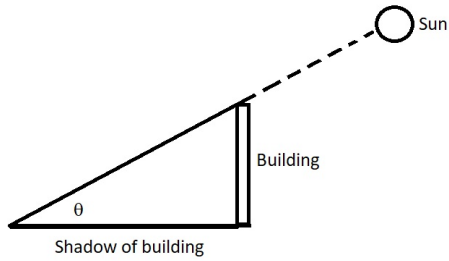
A. 100 ft/hour

B. 200 ft/hour

C. 300 ft/hour

D. 400 ft/hour

E. 500 ft/hour

12. Use a linear approximation to estimate the value of

$$\sin\left(\frac{\pi}{6} + \frac{\pi}{36}\right)$$

A. $\dfrac{\sqrt{3}}{2} + \dfrac{\pi}{72}$

B. $\dfrac{1}{2} + \dfrac{\pi\sqrt{3}}{72}$

C. $\dfrac{\sqrt{3}}{2} + \dfrac{\pi}{60}$

D. $\dfrac{1}{2} + \dfrac{\pi\sqrt{3}}{60}$

E. $\dfrac{\sqrt{3}}{2} + \dfrac{\pi}{36}$

13. The minimum value of $f(x) = x + \dfrac{1}{x}$ on the interval $[\frac{1}{2}, 2]$ is

A. 2

B. $\frac{5}{2}$

C. -2

D. 0

E. $\frac{1}{2}$

14. The Mean Value Theorem guarantees that the derivative of $f(x) = \sqrt{1 + x^3}$ at some point on the interval $(0, 2)$ is

A. 0

B. 1

C. 2

D. 3

E. 4

15. Evaluate the limit $\lim\limits_{x \to 0^+} \left(\dfrac{1}{e^x - 1} - \dfrac{1}{x} \right)$

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

16. The area of the largest rectangle that can be inscribed in the region under the curve $y = e^{-x^2}$ in the first quadrant is

A. $\sqrt{\dfrac{2}{e}}$

B. $\sqrt{2e}$

C. $\dfrac{2}{e}$

D. $\dfrac{1}{\sqrt{2e}}$

E. $\dfrac{2}{e^2}$

17. Find the minimum distance between the line $y = -2x + 5$ and the origin

A. $\sqrt{2}$

B. $\sqrt{3}$

C. $\sqrt{5}$

D. 2

E. 5

18. If $f''(x) = 2x$ and $f(0) = 4$, $f'(0) = -3$, find $f(3)$

A. 0

B. 2

C. 4

D. 7

E. 9

19. $\int \sqrt{x}(\sqrt{x} - 1)\, dx=$

A. $2(x^{3/2} - x) + C$

B. $\frac{1}{2}x^2 - x + C$

C. $\frac{1}{2}(\sqrt{x} - 1)^2 + C$

D. $\frac{1}{2}x^2 - \frac{2}{3}x^{3/2} + C$

E. $x - 2\sqrt{x} + C$

20. Find $\displaystyle\int_0^1 (x - \sqrt{1 - x^2})\, dx.$

Hint: interpret this integral in terms of areas of basic shapes.

A. $\dfrac{1}{2} - \dfrac{\pi}{4}$

B. $1 - \dfrac{\pi}{2}$

C. 0

D. $\dfrac{\pi}{2} - 1$

E. $\dfrac{\pi}{4} - \dfrac{1}{2}$

21. Find the derivative of

$$g(x) = \int_{\cos x}^{1} t^2 \, dt$$

A. $\cos^2 x \sin x$

B. $-\cos^2 x \sin x$

C. $2 \cos x \sin x$

D. $-2 \cos x \sin x$

E. $-2 \cos x$

22. Find the integral:

$$\int_0^1 \frac{2x^3}{\sqrt{3x^4 + 1}} \, dx$$

A. $\dfrac{1}{12}$

B. $\dfrac{1}{6}$

C. $\dfrac{1}{4}$

D. $\dfrac{1}{3}$

E. $\dfrac{1}{2}$

23. Find the integral:
$$\int_0^{\pi/6} \sin^3 3x \cos 3x \, dx$$

A. $\dfrac{1}{12}$

B. $\dfrac{1}{6}$

C. $\dfrac{1}{4}$

D. $\dfrac{1}{3}$

E. $\dfrac{1}{2}$

24. Find the integral:
$$\int_0^{\pi/4} \sin x \sec^3 x \, dx$$

A. $\dfrac{1}{12}$

B. $\dfrac{1}{6}$

C. $\dfrac{1}{4}$

D. $\dfrac{1}{3}$

E. $\dfrac{1}{2}$

25. Find the integral:

$$\int_{e}^{e^9} \frac{dx}{x\sqrt{\ln x}}$$

A. 1

B. 2

C. 3

D. 4

E. 6