MA 16100

FINAL EXAM Green

December 16, 2015

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ RECITATION TIME ____________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. Be sure the paper you are looking at right now is GREEN!

3. Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes):

0 1

4. On the mark-sense sheet, fill in your TA's name and the course number.

5. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

6. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

7. Sign the mark–sense sheet.

8. Fill in your name, etc. on this paper (above).

9. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets.

10. Turn in both the mark–sense sheets and the question sheets when you are finished.

11. If you finish the exam before 9:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20. If you don't finish before 9:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

12. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

1. Find the limit.

$$\lim_{t\to 0} \frac{\sqrt{t^2+16}-4}{t^2}$$

A. 0

B. $\frac{1}{8}$

C. $\frac{1}{4}$

D. ∞

E. -4

2. Where is the function $\frac{\ln x}{x^2-1}$ continuous?

A. $(0,1)\cup(1,\infty)$

B. $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

C. $(0,\infty)$

D. $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$

E. $(-1,1)$

3. If an item is thrown upward on Mars, with velocity 10 m/s, its height in meters after t seconds is given by $y = 10t - 2t^2$. Find the average velocity over the time interval $[1, 3]$.

A. 1 m/s

B. 20 m/s

C. 2 m/s

D. 0 m/s

E. 10 m/s

4. Find the approximate value of $\ln(0.95)$ by considering a linearization.

A. 0.95

B. -0.05

C. $-\dfrac{1}{19}$

D. -0.95

E. $\dfrac{20}{19}$

5. Suppose $f(x) = e^{x^2}$ Find $f''(x)$.

A. $4x^2e^{x^2}$

B. $8x^2e^{x^2}$

C. $2xe^{2x}$

D. $(2+2x)e^{x^2}$

E. $(2+4x^2)e^{x^2}$

6. Find the slope of the tangent line to $f(x) = \dfrac{1}{1+\sqrt{x}}$ at $x = 1$.

A. $-\dfrac{1}{8}$

B. 2

C. $-\dfrac{1}{4}$

D. $\dfrac{1}{2}$

E. 1

7. f is a function with $f(1) = 2$, $f'(1) = 3$, $f\left(\frac{\pi}{4}\right) = 1$, and $f'\left(\frac{\pi}{4}\right) = 4$. If

$$h(x) = f(\tan x),$$

find $\frac{dh}{dx}$ at $x = \frac{\pi}{4}$.

A. 6

B. 8

C. 3

D. 4

E. 2

8. Find the equation of the tangent line to the curve $2y^4 - x^2y = x^3$ at the point $(1, 1)$.

A. $y = \frac{1}{3}x + \frac{2}{3}$

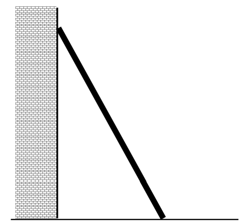
B. $y = \frac{5}{7}x + \frac{2}{7}$

C. $y = \frac{5}{8}x + \frac{3}{8}$

D. $y = x$

E. $y = \frac{5}{9}x + \frac{4}{9}$

9. A ladder of length 10 m is resting against a vertical wall. If the foot of the ladder slides away from the wall at a rate of 3 m/s, how fast is the top of the ladder sliding down the wall at the instant the foot of the ladder is 8m from the wall?



A. $\frac{25}{3}$ m/s

B. 3 m/s

C. 4 m/s

D. $\frac{8}{3}$ m/s

E. $\frac{15}{2}$ m/s

10. Where is the function $f(x) = x^4 - 2x^3 + 4x$ concave down?

A. $(-\infty, 0)$

B. $(-\sqrt{2}, \sqrt{2})$

C. $(1, \infty)$

D. $(0, 1)$

E. $(-\infty, \sqrt{2})$

11. Find the horizontal asymptotes of

$$f(x) = \frac{e^x + 2e^{-x}}{e^x + 3e^{-x}}.$$

A. $y = -1$ and $y = 1$

B. $y = -\dfrac{2}{3}$ and $y = \dfrac{2}{3}$

C. $y = 1$

D. $y = \dfrac{2}{3}$ and $y = 1$

E. There are no horizontal asymptotes.

12. Find the limit.

$$\lim_{x \to 0} \frac{\csc(2x)}{\cot x}$$

A. $\dfrac{1}{2}$

B. 1

C. 2

D. ∞

E. The limit does not exist.

13. Find the vertical asymptotes of the function $f(x) = \dfrac{x^2+1}{3x-2x^2}$.

A. $x = 2$ and $x = -3$

B. $x = -\dfrac{1}{2}$

C. $x = \dfrac{3}{2}$

D. $x = 0$ and $x = \dfrac{3}{2}$

E. $x = \dfrac{1}{3}$

14. Let f be a function whose derivative, f', is given by

$$f'(x) = x(x-2)(x-1)^2.$$

The function f has

A. local maxima at $x = 0$ and $x = 2$.

B. a local minimum at $x = 2$, and a local maximum at $x = 0$.

C. local minima at $x = 0$ and $x = 1$, and a local maximum at $x = 2$.

D. local minima at $x = 0$ and $x = 2$, and a local maximum at $x = 1$.

E. a local minimum at $x = 1$, and local maxima at $x = 0$ and $x = 2$.

15. If 1200 m^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

A. $2000\sqrt{2}\ \text{m}^3$

B. $2000\ \text{m}^3$

C. $2000\sqrt{3}\ \text{m}^3$

D. $1000\sqrt{2}\ \text{m}^3$

E. $4000\ \text{m}^3$

16. Find the point on the curve $y = \sqrt{x}$ which is closest to $(3, 0)$.

Hint: The distance between two points (a, b) *and* (x, y) *is* $\sqrt{(x-a)^2 + (y-b)^2}$.

A. $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$

B. $(1, 1)$

C. $(3, \sqrt{3})$

D. $\left(\frac{3}{2}, \sqrt{\frac{3}{2}}\right)$

E. None of the above.

17. Find an antiderivative of the function $f(x) = \dfrac{2}{1+x^2}$.

A. $2\tan^{-1} x$

B. $\ln(1+x^2)$

C. $2x - \dfrac{2}{x}$

D. $\dfrac{-2}{1+x}$

E. $\dfrac{-4x}{(1+x^2)^2}$

18. Using the table of values below, estimate $\displaystyle\int_1^{2.5} g(x)\, dx$ using a *left* Riemann sum with $n = 3$ rectangles.

x	1	1.5	2	2.5
$g(x)$	3	5	4	2

A. 12

B. 14

C. $\dfrac{28}{3}$

D. $\dfrac{37}{2}$

E. 6

19. Suppose

$$f(x) = \int_0^{x^3} (t^2 + 1)\, dt.$$

Find $f'(x)$.

A. $3x^4 + 3x^2$

B. $3x^8 + 3x^2$

C. $x^6 + 1$

D. $6x^5$

E. $(x^2 + 1)^3$

20. Evaluate.

$$\int_1^4 \frac{\sqrt{x} + x}{x^2}\, dx$$

A. $1 + \ln 16$

B. $-3 + \ln 4$

C. $\dfrac{31}{32} + \ln 16$

D. $\dfrac{3}{2} + \ln 16$

E. $1 + \ln 4$

21. $\displaystyle\int x^3(1-2x^4)^{1/4}\,dx$

A. $-\frac{5}{32}(1-2x^4)^{5/4}+C$
B. $-\frac{1}{8}(1-2x^4)^{5/4}+C$
C. $\frac{4}{5}(1-2x^4)^{5/4}+C$
D. $-\frac{1}{10}(1-2x^4)^{5/4}+C$
E. $\frac{2}{5}(1-2x^4)^{5/4}+C$

22. $\displaystyle\int_{\ln\sqrt{3}}^{\ln\sqrt{8}} 2e^{2t}\sqrt{1+e^{2t}}\,dt$

A. $\dfrac{38}{3}$
B. $\dfrac{32}{3}\sqrt{2}-2\sqrt{3}$
C. $\dfrac{1}{12}$
D. $2\sqrt{2}-\sqrt{3}$
E. $\dfrac{8\ln(3)-8}{5}$

23. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{3\pi}{4}} (\cos t + \sin t)\, dt$

A. $-\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}$

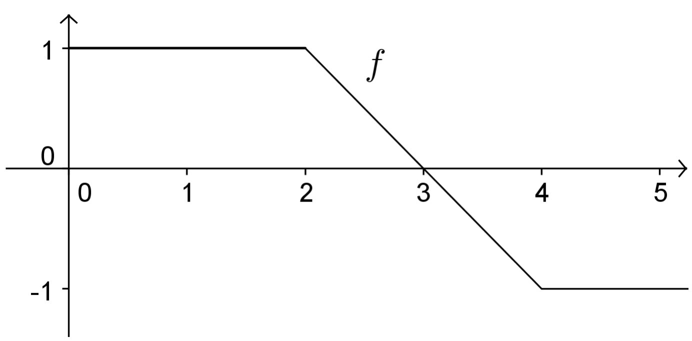
B. $\sqrt{2} - \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}$

C. $\sqrt{2} + \dfrac{\sqrt{3}}{2} - \dfrac{1}{2}$

D. $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}$

E. $-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}$

24. The graph of f is pictured below.



Suppose $F(x) = \displaystyle\int_0^x f(t)\, dt$. Find $F(5)$.

A. -1

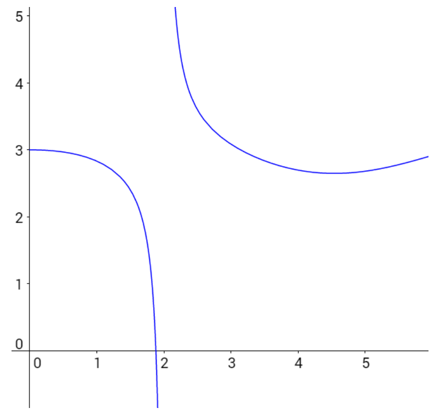
B. 0

C. 1

D. 4

E. -2

25. Here is the graph of f:



Find the graph of its derivative, f', below.