MA 16100
FINAL EXAM Form A (Test/Quiz Number 01)
December 16, 2014

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

1. You must use a #2 pencil on the mark sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

6. Sign the mark sense sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1-25. Do all your work on the question sheets.

9. Turn in both the mark sense sheets and the question sheets when you are finished.

10. If you finish the exam before 5:20, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 5:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

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EXAM POLICIES

1. Students may not open the exam until instructed to do so.

2. Students must obey the orders and requests by all proctors, TAs, and lecturers.

3. No student may leave in the first 20 min or in the last 10 min of the exam.

4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.

5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.

6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. Find the constants a and b such that

$$F(x) = \begin{cases} \dfrac{x^2 - x}{x - 1}, & x < 1 \\ ax^2 + bx + 1, & 1 \le x < 2 \\ x + a, & x \ge 2 \end{cases}$$

is continuous for all x

A. $a = 3, b = -1$

B. $a = 1, b = 1$

C. $a = -1, b = 3$

D. $a = 1, b = -1$

E. $a = 2, b = 0$

2. What is the domain of $f(x) = \sqrt{4x - x^2} + \ln(1 - x)$?

A. $[0, 1) \cup (1, 4]$

B. $[0, 1)$

C. $(1, 4]$

D. $[1, 4]$

E. $(0, 1]$

3. If $f(x) = \dfrac{1}{x+3}$, then $\displaystyle\lim_{x\to 1}\dfrac{f(x)-f(1)}{x-1} =$

A. $-1/16$

B. $1/16$

C. $-1/4$

D. $1/4$

E. Does not exist

4. Determine the limit

$$\lim_{x\to -1^-}\frac{1-x}{x^2(x+1)^3}.$$

A. -2

B. -1

C. 1

D. ∞

E. $-\infty$

5. A sample of a radioactive element initially has mass of 24 gm. After 2 minutes the sample of that element has mass of 2 gm. When (in minutes) is the mass equal to 4 gm?

A. $\dfrac{2\ln 6}{\ln 12}$

B. $\dfrac{\ln 6}{\ln 12}$

C. $\dfrac{2\ln 12}{\ln 6}$

D. $\dfrac{3\ln 6}{2\ln 2}$

E. $\dfrac{3\ln 8}{\ln 3}$

6. Compute the limit

$$\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}.$$

A. $-\frac{1}{6}$

B. 1

C. -1

D. $\frac{1}{6}$

E. The limit does not exist.

7. For what values of x does the graph of $f(x) = \dfrac{\ln(x)}{x}$ have a horizontal tangent line?

A. The graph does not have a horizontal tangent line.

B. e

C. 1

D. $2/e$

E. $1/e$

8. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

A. -4

B. -2

C. -1

D. 0

E. 2

9. Which of the following is/are true about the function $g(x) = 4x^3 - 3x^4$?

   (1) g is decreasing for $x > 1$

   (2) g has a relative extreme value at $(0,0)$

   (3) the graph of g is concave up for all $x < 0$

   A. (1), (2) and (3)
   B. only (1)
   C. only (2)
   D. (1) and (2)
   E. (1) and (3)

10. The slope of the line tangent to the graph of

$$y^2 - 2x^3 + xy^4 + 13 = 0$$

   at $(2,1)$ is

   A. $\dfrac{23}{10}$

   B. $\dfrac{12}{5}$

   C. $\dfrac{18}{5}$

   D. $-\dfrac{5}{18}$

   E. -13

11. The position of a particle moving along the x-axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \geq 0$. When $t = \pi$, the acceleration is

A. 9

B. 1/9

C. 0

D. $-1/9$

E. -9

12. Use a linear approximation to estimate $e^{-0.01}$.

A. 0.9

B. 0.99

C. 0.999

D. 1.01

E. 1.001

8

13. Find the absolute maximum value of $f(x) = \dfrac{x+1}{x^2+3}$ on the interval $[-2, 3]$.

A. 1/3

B. 3/7

C. $(\sqrt{2}+1)/5$

D. 8/13

E. 1/2

14. If c is a number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval $[0, 2]$, then $c =$

A. 0

B. 1/2

C. 4/3

D. 1

E. 2

15. How many inflection points does the function $f(x) = x^4 - 4x^2$ have?

    A. 0

    B. 1

    C. 2

    D. 3

    E. 4

16. Evaluate $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1} \right)$

    A. ∞

    B. 0

    C. $\dfrac{1}{2}$

    D. 2

    E. $-\infty$

17. A rectangular cardboard box with no top has a rectangular base so that one side is twice as long as the other. If the box must have a volume of $\frac{4}{3}$ m^3, what should the height of the box be to minimize the amount of cardboard used?

A. $\left(\dfrac{2}{3}\right)^{\frac{1}{3}}$ m

B. $\dfrac{2}{3}$ m

C. $\dfrac{1}{\sqrt{2}}$ m

D. $\dfrac{2\sqrt{2}}{3}$ m

E. $\dfrac{2}{\sqrt[3]{3}}$ m

18. If the graph of $y = \dfrac{ax + b}{x + c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

A. -5

B. -1

C. 0

D. 1

E. 5

19. The graph of a piecewise-linear function f, for $-1 \leq x \leq 4$, is show below. What is the value of $\displaystyle\int_{-1}^{2} f(x)\, dx$?
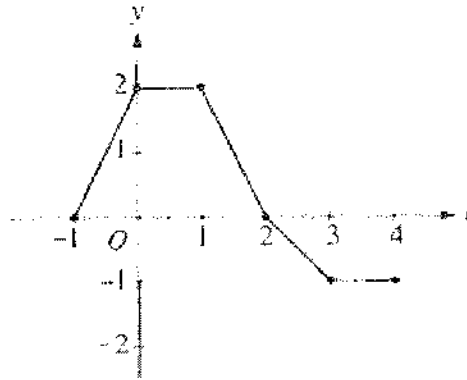
A. 1

B. 2.5

C. 4

D. 5.5

E. 8

20. $\displaystyle\int x^3(1 - 2x^4)^{1/4}dx =$

A. $-\dfrac{5}{32}(1 - 2x^4)^{5/4} + C$

B. $-\dfrac{1}{8}(1 - 2x^4)^{5/4} + C$

C. $\dfrac{4}{5}(1 - 2x^4)^{5/4} + C$

D. $\dfrac{2}{5}(1 - 2x^4)^{5/4} + C$

E. $-\dfrac{1}{10}(1 - 2x^4)^{5/4} + C$

21. Let $h(x) = f(g(x))$, where $g(1) = 2$, $g'(1) = 3$, $f(1) = 4$, $f'(1) = 5$, $f(2) = 6$, and $f'(2) = 7$. Then $h'(1) =$

A. 5

B. 7

C. 15

D. 21

E. None of the above is necessarily correct

22. If $f(x) = \begin{cases} 8 - x^2 & -2 \leq x \leq 2 \\ x^2 & \text{elsewhere} \end{cases}$. Then $\int_{-1}^{3} f(x)\, dx$ is a number between

A. 0 and 8

B. 8 and 16

C. 16 and 24

D. 24 and 32

E. 32 and 40

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23. Evaluate

$$\int_1^e \frac{x^2 - 1}{x}\, dx$$

A. $\dfrac{e^2}{2} - \dfrac{3}{2}$

B. $e - \dfrac{1}{e}$

C. $e^2 - 2$

D. $\dfrac{e^2}{2} - e + \dfrac{1}{2}$

E. $e^2 - 2$

24. Evaluate

$$\int_0^{\pi/2} \frac{\cos(\theta)}{\sqrt{1 + \sin(\theta)}}\, d\theta$$

A. $-2(\sqrt{2} - 1)$

B. $-2\sqrt{2}$

C. $2\sqrt{2}$

D. $2(\sqrt{2} - 1)$

E. $2(\sqrt{2} + 1)$

25. The velocity of a particle moving on a line at time t is $v(t) = 3t^{1/2} + 5t^{3/2}$ meters/second. What distance did it travel, in meters, from $t = 0$ to $t = 4$?

A. 40

B. 32

C. 80

D. 64

E. 184