MA 16100
EXAM 3 Green
April 11, 2016

NAME YOUR TA'S NAME $\qquad$

STUDENT ID \# $\qquad$ RECITATION TIME

1. You must use a $\# 2$ pencil on the mark-sense sheet (answer sheet).
2. Be sure the paper you are looking at right now is GREEN!
3. Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes):
4. On the mark-sense sheet, fill in your TA's name and the course number.
5. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.
6. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.
7. Sign the mark-sense sheet.
8. Fill in your name, etc. on this paper (above).
9. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1-12. Do all your work on the question sheets.
10. Turn in both the mark-sense sheets and the question sheets when you are finished.
11. If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20.
If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.
12. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

## EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME:

STUDENT SIGNATURE:

1. An observer is stationed 300 feet from a rocket launch pad. The rocket rises vertically off the launch pad. A few seconds after takeoff, the rocket is 300 feet in the air and rising at 100 feet $/ \mathrm{sec}$. How fast is the angle of elevation, $\theta$, changing at that instant?

A. $\frac{1}{6}$ radian $/ \mathrm{sec}$
B. 2 radian $/ \mathrm{sec}$
C. $\frac{1}{2}$ radian $/ \mathrm{sec}$
D. $\frac{1}{4}$ radian $/ \mathrm{sec}$
E. $\frac{2}{3}$ radian $/ \mathrm{sec}$
2. Use a linear approximation (or differentials) to estimate the value of $\sqrt{24.8}$
A. 5.20
B. 4.98
C. 4.95
D. 4.92
E. 4.90
3. Find the minimum value of $f(x)=x^{3}-x$ on the closed interval $[-1,1]$.

Hint: Find the actual value of $f$ and NOT the $x$-value at which that minimum occurs.
A. 0
B. $-\frac{1}{\sqrt{3}}$
C. $-\frac{1}{3}$
D. $-\frac{2}{3 \sqrt{3}}$
E. There is no absolute minimum value.
4. The function $f$ is continuous on $[0,2]$ and differentiable on $(0,2)$, and consequently the direct application of the mean value theorem guarantees the existence of $c$, where $c$ is between 0 and 2 and pictured below. Find $f^{\prime}(c)$.

A. -2
B. 1.2
C. 0.5
D. -4
E. -1
5. Which statement accurately describes the function

$$
f(x)=x^{4}-6 x^{3}
$$

on the interval $(0,3)$ ?
A. $f$ is increasing and its graph is concave up.
B. $f$ is decreasing and its graph is concave up.
C. $f$ is increasing and its graph is concave down.
D. $f$ is decreasing and its graph is concave down.
E. None of the above.
6. The graph of $y=f^{\prime}(x)$, the derivative of $f$, is shown below.


Which of the following statements about $f$ are true?
I. The graph of $f$ is concave up on the interval $(2,4)$.
II. $f(x)$ has a local minimum at $x=2$.
III. $(1, f(1))$ is an inflection point for $f$.
A. None of these statements are true.
B. I and III
C. II and III
D. I and II
E. I, II, and III
7. Find the limit.

$$
\lim _{x \rightarrow 0} \frac{\tan x-x}{x^{3}}
$$

A. $-\infty$
B. $\frac{1}{3}$
C. 1
D. $-\frac{1}{6}$
E. 0
8. Find the $x$-coordinate of the inflection point of the function $f(x)=\frac{1}{\ln x}$ on the interval $0<x<1$.
A. $x=\frac{1}{2}$
B. $x=\frac{1}{e^{2}}$
C. $x=\frac{1}{\sqrt{e}}$
D. $x=\frac{1}{e}$
E. $x=\ln 2$
9. A six-sided box is to have four clear plastic sides, a wooden square top, and a wooden square bottom. The volume of the box must be $24 \mathrm{ft}^{3}$. Plastic costs $\$ 1 \mathrm{per} \mathrm{ft}^{2}$ and wood costs $\$ 3$ per $\mathrm{ft}^{2}$. Find the dimensions of the box which minimize cost.

A. $2 \mathrm{ft} \times 2 \mathrm{ft} \times 6 \mathrm{ft}$
B. $\sqrt{6} \mathrm{ft} \times \sqrt{6} \mathrm{ft} \times 4 \mathrm{ft}$
C. $\sqrt[3]{4} \mathrm{ft} \times \sqrt[3]{4} \mathrm{ft} \times 6 \sqrt[3]{4} \mathrm{ft}$
D. $\sqrt[3]{3} \mathrm{ft} \times \sqrt[3]{3} \mathrm{ft} \times 8 \sqrt[3]{3} \mathrm{ft}$
E. $2 \sqrt[3]{2} \mathrm{ft} \times 2 \sqrt[3]{2} \mathrm{ft} \times 3 \sqrt[3]{2} \mathrm{ft}$
10. A rectangle is formed with one corner at $(0,0)$ and the opposite corner on the graph of $y=-\ln x$, where $0<x<1$. What is the largest possible area of such a rectangle?

A. $\frac{\sqrt{e}}{2}$
B. $e$
C. $\frac{1}{e}$
D. $\frac{\ln 2}{2}$
E. There is no maximum.
11. Suppose $f$ is a differentiable function with $f^{\prime \prime}(x)>0$ for all real numbers $x$. Assume that $f(1)=3$ and $f(5)=3$. Which one of these statements must be true?
A. $f^{\prime}(x)$ is decreasing at $x=3$.
B. $f(x) \geq 0$ for all real numbers $x$.
C. $f$ has an inflection point.
D. $f^{\prime}(3)>0$.
E. $f$ has a local minimum.
12. Which of these curves is the graph of $y=5 x^{6}+6 x^{5}$ ?


