MA 16100
EXAM 3 (Version 11)
(November 18, 2021)

NAME _____ YOUR TA's NAME _____

PUID # _____ RECITATION TIME _____

Write the following in the TEST/QUIZ NUMBER boxes: 11 (and blacken the appropriate digits below the boxes). You must use a #2 pencil on the mark-sense sheet (answer sheet). On the mark-sense sheet, fill in your TA's NAME and the COURSE number. Fill in YOUR NAME and PUID NUMBER and blacken the appropriate spaces. Fill in your four-digit SECTION NUMBER. If you do not know your section number, ask your TA. Sign the mark-sense sheet.

There are **12** questions, each worth 8 points. You automatically earn 4 points for taking the exam, for a total of 100 points. Blacken in your choice of the correct answer in the spaces provided for questions 1-12. Do all your work in the exam booklet, and also circle the answers in the exam booklet in case of a lost mark-sense sheet. Use the back of the test pages for scrap paper. Turn in both the mark-sense sheet and the exam booklet when you are finished.

If you finish the exam before 8:50pm you may leave the room after turning in the mark-sense sheet and exam booklet. You may not leave the room before 8:20pm. If you don't finish before 8:50pm you MUST REMAIN SEATED until your TA comes and collects your mark-sense sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so
2. Students must obey the orders and requests by all proctors, TA, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the mark-sense sheets and exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME _____

STUDENT SIGNATURE _____

1. A 5-foot ladder is leaning against a wall. A person pushes the base of the ladder toward the wall at a rate of 3 ft/min. When the base of the ladder is 3 ft from the wall, how fast is the top of the ladder moving up the wall?

   A. $\frac{9}{4}$ ft/min
   B. $\frac{4}{3}$ ft/min
   C. 9 ft/min
   D. $\frac{3}{4}$ ft/min
   E. 4 ft/min

2. An inverted conical water tank (that is, the point of the cone is pointing down) has base radius 4 ft and height 12 ft. Water is pumped into the cone at a rate of 36π ft^3/min. How fast is the water level rising at the time when the water in the cone is 2 ft deep? *Hint:* The volume of a cone is $V = \frac{1}{3}\pi r^2 h$

   A. $\frac{27}{4}$ ft/min
   B. 1 ft/min
   C. $\frac{9}{4}$ ft/min
   D. 9 ft/min
   E. 81 ft/min

3. Suppose $f(x) = \cos^2(x) + \sin(x)$. If M is the absolute maximum of f on the interval $[0, \frac{\pi}{2}]$ and m is the absolute minimum of f on the same interval, what is $M + m$?

A. $\frac{13}{4}$

B. $\frac{7}{4}$

C. $\frac{5}{4}$

D. $\frac{9}{4}$

E. $\frac{11}{4}$

4. Find the number c that satisfies the conclusion of the Mean Value Theorem for the function $f(x) = 3 \ln x$ on the interval $[1, e^2]$

A. $\frac{e^2 - 1}{6}$

B. $\frac{e^2 - 1}{2}$

C. $\left(\frac{e^2 - 1}{6}\right)^{1/3}$

D. $\frac{e^2 - 1}{3}$

E. $\left(\frac{e^2 - 1}{3}\right)^{1/3}$

5. If the **second derivative** of $f(x)$ is

$$f''(x) = \frac{(x+1)^2(x-3)(x+5)^3(x-6)^4}{x^2(x-1)^2},$$

how many inflection points does the graph of $y = f(x)$ have?

A. 1

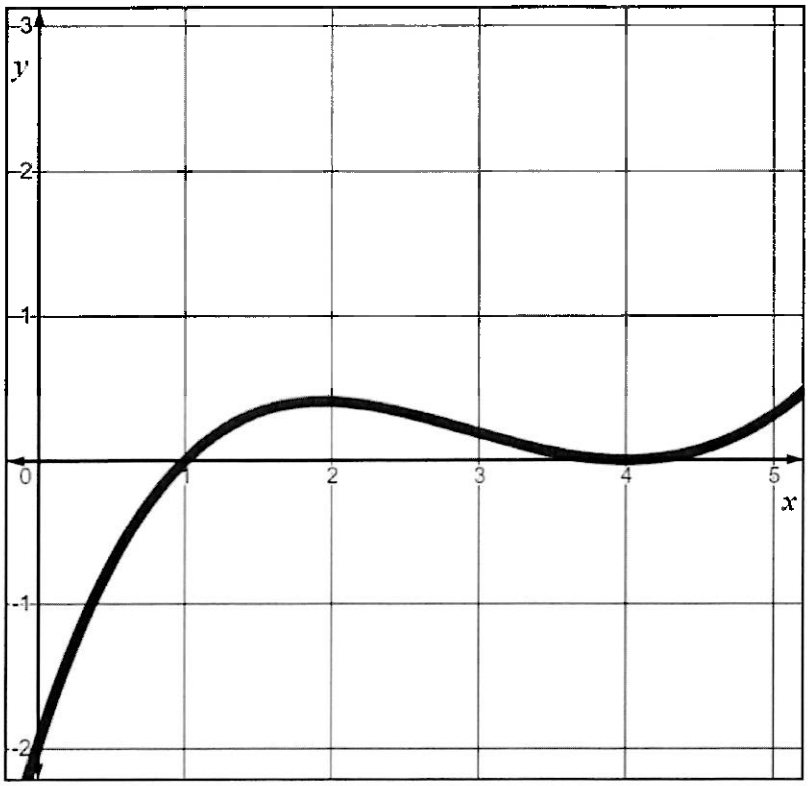
B. 2

C. 5

D. 4

E. 3

6. The graph of the **first derivative** $f'(x)$ of a function $f(x)$ is shown below. Which of the following statements are true?



I. $f(x)$ has a local minimum at $x = 1$

II. $f(x)$ decreasing on $(2, 4)$

III. $f(x)$ is concave downward on $(0, 1)$

A. II and III

B. I and II

C. I only

D. I and III

E. II only

7. If x and y are positive numbers with $xy = 54$, what is the smallest value $3x + 2y$ can have?

   A. 45

   B. 30

   C. 36

   D. 39

   E. 42

8. A cylindrical barrel has radius r and height h and total volume 32π m^3. The top and bottom of the barrel are made of wood that costs \$2 per m^2 and the sides are made of plastic that costs \$1 per m^2. Find the radius of the barrel with the smallest cost. (Hint: the volume of a cylinder is $V = \pi r^2 h$)

   A. 8

   B. 1

   C. 4

   D. 2

   E. 6

9. Use linear approximation and the fact that $(1000)^{1/3} = 10$ to estimate $(1006)^{1/3}$.

A. 10.06
B. 10.20
C. 10.09
D. 10.01
E. 10.02

10. Find the limit:

$$\lim_{x \to 0} \frac{e^{3x^2} - 1}{\cos(x) - 1}$$

A. -6
B. 6
C. 0
D. 3
E. -3

11. Find the limit:

$$\lim_{x \to 0^+} (1 + 3x)^{2/x}$$

A. e^4

B. e^3

C. e

D. e^2

E. e^6

12. For the function

$$f(x) = x^{-1/3} + x^{-1/2}$$

let $F(x)$ be the antiderivative that satisfies $F(1) = 5$. Find $F(0)$.

A. $\frac{5}{2}$

B. $-\frac{1}{2}$

C. $-\frac{5}{2}$

D. $\frac{3}{2}$

E. $\frac{1}{2}$