MA 161 & 161E Midterm Exam 3, November 2003

Name _____

Student ID number _____

Lecturer _____

Recitation instructor _____

INSTRUCTIONS:

1. Fill in all the information requested above and on the scantron sheet.
2. This booklet contains 12 problems, each worth 8 points. You get 2 points for coming and 2 if you fully comply with instruction 1. The maximum score is 100 points.
3. For each problem circle the answer of your choice, and also mark it on the scantron sheet.
4. Work only on the pages of this booklet.
5. Books, notes, calculators are not to be used on this test.
6. At the end turn in your exam and scantron sheet to your recitation instructor.

Formulas (which you may or may not use)

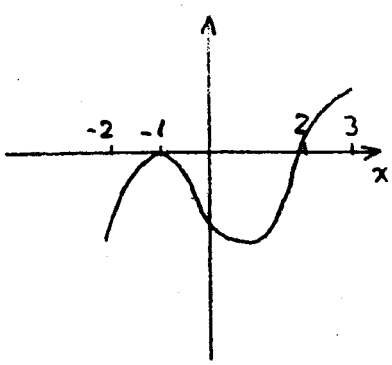
$$\cos 2x = \cos^2 x - \sin^2 x, \qquad \sin 2x = 2 \sin x \cos x,$$

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \quad \text{(law of sines)},$$

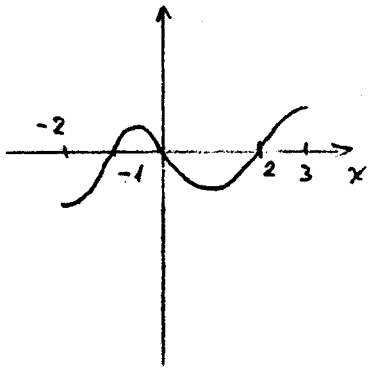
$$c^2 = a^2 + b^2 - 2ab \cos \gamma \quad \text{(law of cosines)}$$

$$A = \frac{h}{2}(l_1 + l_2) \quad \text{(area of trapezium)}.$$

1. Given that $f'(x) > 0$ when $-1 < x < 0$ and $2 < x < 3$, and $f'(x) < 0$ when $-2 < x < -1$ and $0 < x < 2$, which could be the graph of f?



A.



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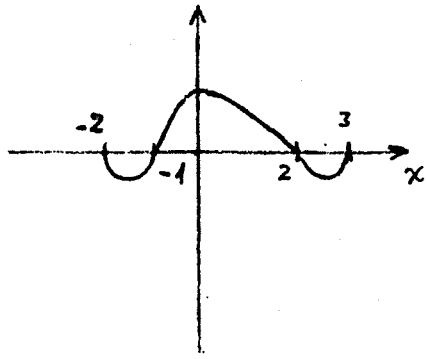
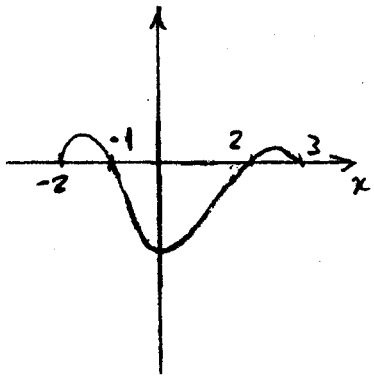
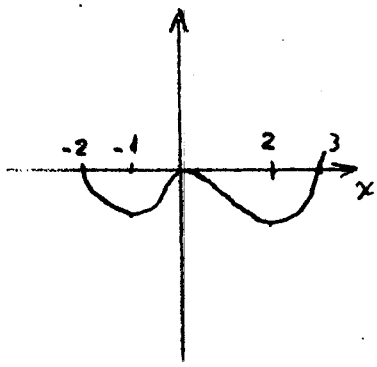


C.



D.



E.

2. The derivative of a function g is $g'(x) = \sin x - \sin 2x$, so that $x = 0$ and $x = \pi/3$ are critical numbers of g. Then, g has

   A. a local minimum at 0 and a local maximum at $\pi/3$

   B. a local maximum at 0 and a local minimum at $\pi/3$

   C. a local maximum at 0 and an inflection point at $\pi/3$

   D. a local maximum at $\pi/3$

   E. inflection points at $0, \pi/3$

3. $\lim_{x\to\infty} \frac{\ln x}{e^{2x}} =$

A. ∞

B. e

C. 1

D. 0

E. -1

4. $\lim_{x\to 0} \frac{1-\cos 2x}{x^2} =$

A. 0

B. ∞

C. $\frac{\pi}{2}$

D. 1

E. 2

5. $\lim_{x \to 0^+} (1 + \sin x)^{1/x} =$

A. 0

B. ∞

C. $\ln 2$

D. 2

E. e

6. The minimum value of $f(x) = 3x + \frac{12}{x^2}$ for $x > 0$ is

A. 6

B. 8

C. $\frac{26}{3}$

D. 9

E. 10

7. The minute hand on a watch is 2 in long and the hour hand is 1 in long. At two o'clock the distance between the tips of the hands is $\sqrt{3}$ in. How fast is the distance between the tips of the hands decreasing at that moment?

A. $\frac{11\pi}{6}$ in/hour

B. $\frac{11\pi\sqrt{3}}{6}$ in/hour

C. $\frac{11\pi}{12}$ in/hour

D. $\frac{11\pi\sqrt{3}}{12}$ in/hour

E. $\frac{11\pi}{6\sqrt{3}}$ in/hour

8. The linear approximation of $f(x) = x^{20}$ at $a = 20$ is used to find an approximate value for 19^{20}. The approximate value found is

A. 19^{19}

B. 19^{20}

C. -19^{19}

D. 20^{19}

E. 0

9. Suppose that f is continuous on $[2,5]$ and $2 \leq f'(x) \leq 5$ for all x in $(2,5)$. Then, the mean value theorem implies that $f(5) - f(2)$ lies in the interval

A. $[6,15]$

B. $[3,12]$

C. $[2,5]$

D. $[0,5]$

E. $[-5,5]$

10. The critical numbers of $R(t) = t^{1/3} - t^{-2/3}$ are

A. 0 and 2

B. -2 only

C. 0 and $\pm\sqrt{3}$

D. -2 and -1

E. 2 and $\pm\sqrt{3}$

11. A rain gutter is to be constructed from a metal sheet of width 24 cm by bending up one-third of the sheet on each side through an angle θ. In order to choose θ so that the gutter will carry the maximum amount of water, the function to be maximized is

A. $64\,(\cos^2\theta + \cos\theta)$

B. $64\,(\sin\theta\,\cos\theta + \sin\theta)$

C. $32\sin^2\theta + 16\cos^2\theta$

D. $32\,(\sin^2\theta + \sin\theta\,\cos\theta)$

E. $32\cos^2\theta + 16\sin\theta\,\cos\theta$

12. The total number of local maxima, local minima, and inflection points in the graph of $f(x) = \dfrac{1}{1-x^2}$ is

A. 1

B. 2

C. 3

D. 4

E. 5