MA 16100
EXAM 1 Version A
February 7, 2023

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Write your final answers in the boxes provided, as applicable.

The problems are numbered 1–11.

Extra scratch paper is not permitted. Write all your work in this exam booklet.

If you finish the exam before 7:20, you may leave the room after turning in the exam booklet. If you don't finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your exam booklet. You may not leave the room before 6:50.

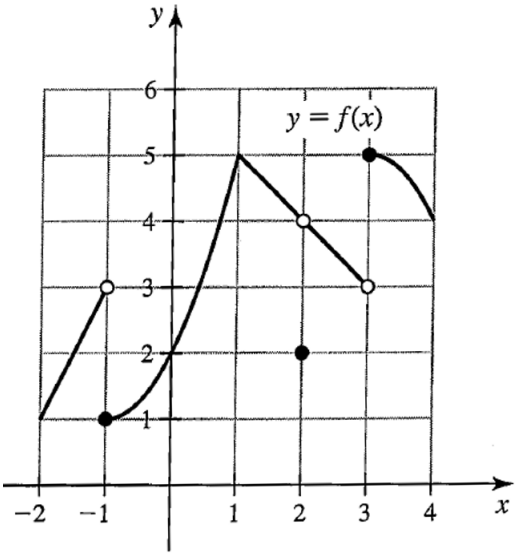
EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: _____

1. Use the graph of $y = f(x)$ to determine the value of each limit below.
   Work will not be considered; only your final answer will be evaluated for credit.
   If a limit does not exist, then write "D.N.E."



(a) (4 points)

$\lim_{x \to 2} f(x) =$

(b) (4 points)

$\lim_{x \to -1^+} f(x) =$

(c) (4 points)

$\lim_{x \to 3} f(x) =$

2. Compute the following limits.
   If the limit is ∞ or $-\infty$ then write that symbol as your final answer.
   If the limit does not exist, and is neither ∞ nor $-\infty$, then write "D.N.E."
   Work will not be considered; only your final answer will be evaluated for credit.

   (a) (5 points)

   $$\lim_{x \to 1} \frac{x - 1}{\sqrt{3x + 6} - 3}$$

   (b) (5 points)

   $$\lim_{x \to -5^+} \frac{x^2}{x^2 - 25}$$

   (c) (5 points)

   $$\lim_{x \to \infty} \frac{(2x + 2)(1 + x)^2}{(x^2 + 1)(1 - 3x)}$$

3. Consider the graph of $y = \dfrac{\cos x \, \sin x}{e^x}$.

(a) (4 points) List all the vertical asymptotes. If there are none, write "none." Work will not be considered; only your final answer will be evaluated for credit.

(b) (4 points) List all the horizontal asymptotes. If there are none, write "none." Work will not be considered; only your final answer will be evaluated for credit.

4. (9 points) Assume f is an odd, continuous function with these values:

x	1	2	3
$f(x)$	-3	3	1

On which of the following intervals does the equation

$$f(x) = 0$$

have at least one solution?

Hint: Use the Intermediate Value Theorem.
Work will not be considered; only your final answer will be evaluated for credit.

More than one of these intervals may be correct. Circle **all** correct answers below:

$-3 \leq x \leq -2$ $\quad -2 \leq x \leq -1$ $\quad -1 \leq x \leq 1$ $\quad 1 \leq x \leq 2$ $\quad 2 \leq x \leq 3$

5. (8 points) Find **all** solutions with $0 \leq x \leq 2\pi$.

$$\sin 2x = -\frac{\sqrt{3}}{2}$$

x is one of these values:

6. (8 points) If $\log_b x = 4$ and $\log_b \left(\dfrac{x^3}{\sqrt{y}} \right) = 2$, find $\log_b y$. Show your work.

$\log_b y =$

7. (8 points)

$$f(x) = \begin{cases} ax^3 & \text{, for } x < -1 \\ \cos^{-1} x & \text{, for } -1 \leq x \leq 1 \end{cases}$$

Find the value of a for which $f(x)$ is continuous for all $x < 1$. Show your work.

$a =$

8. (8 points) An object's position function is given by $s(t) = -t^2 + 6t + 10$, where t is time. Find the **average velocity** of the object over the time interval $[0, h]$ where $h > 0$. Show your work. Simplify your answer completely. *Hint: The answer is a function of h.*

$v_{\text{av}} =$

9. (8 points) Suppose $f(1) = 3$ and the derivative function is $f'(x) = \sec(\ln x)$.
Find the equation for the line tangent to the graph of $y = f(x)$ at $(1, 3)$.
Show your work.

The tangent line equation is

10. (8 points) Use the definition

$$m_{\tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

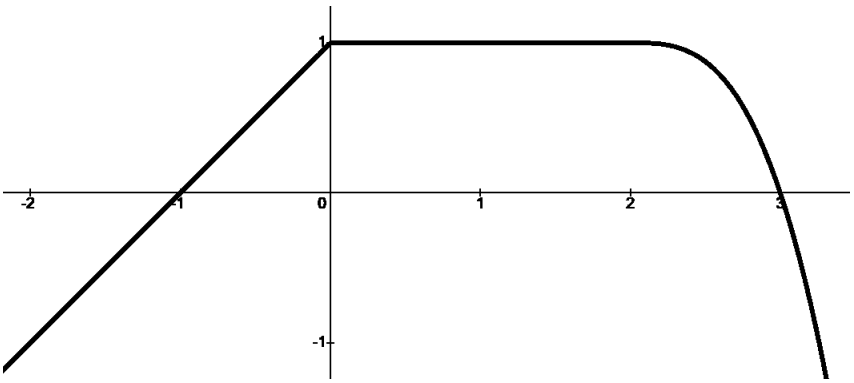
to calculate the slope of the line tangent to the graph of

$$f(x) = x^2 + 2x$$

at the point $(1, 3)$.
Simplify completely. Show each step of your work and do not take shortcuts.

$m_{\tan} =$

11. (8 points) Here is the graph of $y = f(x)$:



On the axes below, carefully sketch the graph of $y = f'(x)$, the derivative of f.