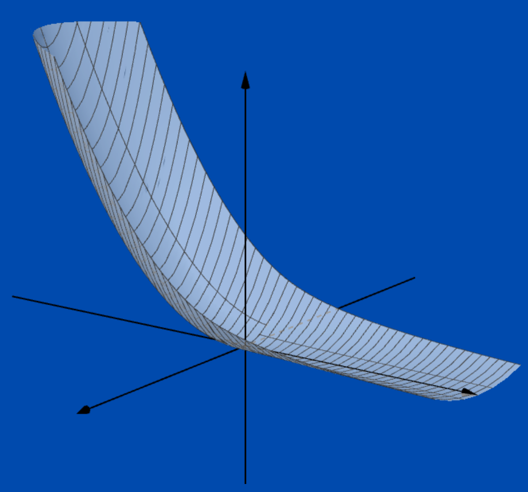
GEOMETRIC AND FUNCTIONAL INEQUALITIES AND RECENT TOPICS IN NONLINEAR PDES



FEBRUARY 28TH & MARCH 1ST, 2021

Invited speakers

Eric Baer, University of Chicago
Shibing Chen, University of Science and Technology of China
Marco Cirant, Università di Padova
Serena Dipierro, University of Western Australia
Cristian Gavrus, Johns Hopkins University
Alessandro Goffi, Università di Padova
Ryan Hynd, University of Pennsylvania
Matthew Jacobs, University of California, Los Angeles
Daesung Kim, University of Illinois at Urbana Champaign
Victor Lie, Purdue University
Alpar Meszaros, Durham University
Robin Neumayer, Northwestern University
Stefania Patrizi, University of Texas at Austin
Xavier Ros-Oton, Universitat de Barcelona
Simon Schulz, University of Cambridge

Organizers

Emanuel Indrei, Purdue University
Diego Marcon, Universidade Federal do Rio Grande do Sul
Levon Nurbekyan, University of California at Los Angeles

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MEETING ID: 944 7884 3244

PASSCODE: 596891

Geometric and functional inequalities and recent topics in nonlinear PDEs

Organizers:
Emanuel Indrei, Purdue University
Levon Nurbekyan, University of California, Los Angeles
Diego Marcon, Federal University of Rio Grande do Sul

Sunday February 28th

Conference Intro: 8:25

8:30-9:20: Xavier Ros-Oton
9:30-10:20: Marco Cirant
10:30-11:20: Simon Schulz
11:30-12:20 Cristian Gavrus
12:20-2:30 Lunch Break
2:30-3:20 Stefania Patrizi
3:30-4:20 Matthew Jacobs
4:30-5:20 Coffee/Tea Break
5:30-6:20 Robin Neumayer

Monday March 1st

8:30-9:20: Alessandro Goffi
9:30-10:20: Ryan Hynd
10:30-11:20: Shibing Chen
11:30-12:20 Alpar Meszaros
12:20-2:30 Lunch Break
2:30-3:20 Victor Lie
3:30-4:20 Daesung Kim
4:30-5:20 Eric Baer
5:30-6:20 Serena Dipierro

1. Shibing Chen, University of Science and Technology of China

   email: chenshibing1982@hotmail.com

   Title: Smoothness of free boundaries in optimal transport problem

   Abstract: Free boundaries arise in optimal transport problem when only a portion of mass is transported. The C^1 and $C^{1,\alpha}$ regularity of free boundaries were established by Caffarelli and McCann when the domains are disjoint, and later by Figalli, Indrei when the domains are allowed to have overlap. We will discuss our recent proof to the higher regularity of free boundary. This is based on a joint work with Jiakun Liu and Xu-Jia Wang.

2. Robin Neumayer, Northwestern University

   email: neumayer@math.northwestern.edu

   Title: Quantitative stability for minimizing Yamabe metrics

   Abstract: The Yamabe problem asks whether, given a closed Riemannian manifold, one can find a conformal metric of constant scalar curvature (CSC). An affirmative answer was given by Schoen in 1984, following contributions from Yamabe, Trudinger, and Aubin, by establishing the existence of a function that minimizes the so-called Yamabe energy functional; the minimizing function corresponds to the conformal factor of the CSC metric.

   We address the quantitative stability of minimizing Yamabe metrics. On any closed Riemannian manifold we show—in a quantitative sense—that if a function nearly minimizes the Yamabe energy, then the corresponding conformal metric is close to a CSC metric. Generically, this closeness is controlled quadratically by the Yamabe energy deficit. However, we construct an example demonstrating that this quadratic estimate is false in the general. This is joint work with Max Engelstein and Luca Spolaor.

3. Victor Lie, Purdue University

   email: vlie@purdue.edu

   Title: The LGC-method

   Abstract: In this talk we will discuss a versatile method developed by the author in order to analyze the boundedness properties of large classes of (sub-)linear or multi-linear operators. More precisely, choosing as a generic example an operator of the form

   $$T(f,g)(x)=\int\int_{\mathbb{R}^2}\widehat{f}(\xi)\,\widehat{g}(\eta)\,m(x,\xi,\eta)\,e^{i\xi x}\,e^{i\eta x}\,d\xi\,d\eta\,,\qquad x\in\mathbb{R}, \tag{1}$$

   having as a symbol

   $$m(x,\xi,\eta)=e^{i\,\varphi(x,\xi,\eta)}\,\psi(x,\xi,\eta)\qquad\text{with }\varphi,\ \psi\text{ real functions, }\ \psi\geq 0, \tag{2}$$

that satisfies *suitable* 1) smoothness in the ξ and η variables–but *not necessarily* in the x variable, 2) non-degeneracy conditions and 3) non-zero curvature in ξ, η one can prove that T obeys natural L^p bounds.

The proof relies on the so called *LGC-method* consisting of three key steps:

- phase *linearization*: the frequency plane is discretized in regions within which the phase of the multiplier φ oscillates at the linear level;
- *Gabor* frame discretization: within each of the regions obtained at the first item, one performs an adapted Gabor frame decomposition of the input functions f, g;
- *cancelation* via time-frequency *correlation*: the resulting discretized operator is now analyzed at the L^2 level via a TT^* argument exploiting the size distribution of the Gabor coefficients via the structure of the time-frequency correlation level sets.

Through the above we provide a new and unified approach to three main themes in Harmonic Analysis:

- The Linear Hilbert Transform and Maximal Operator along variable curves;
- Carleson Type operators in the presence of curvature;
- The bilinear Hilbert transform and maximal operator along variable curves.

Moreover, this method proves to be essential for understanding the behavior of a new class of hybrid operators that have *both zero and non-zero curvature* features:

Theorem 1. *(jointly with C. Benea, F. Bernicot and M. Vitturi, 2021) Let* $a \in (0, \infty) \setminus \{1, 2, 3\}$ *and assume* p, q, r *are Hölder indices, i.e.* $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$, *with* $1 < p, q \leq \infty$ *and* $\frac{2}{3} < r < \infty$. *Then the non-resonant Bilinear Hilbert–Carleson operator defined as*

$$BC^a(f,g)(x) := \sup_{\lambda \in \mathbb{R}} \Big| \int f(x-t)\, g(x+t)\, e^{i\lambda t^a} \, \frac{dt}{t} \Big| \tag{3}$$

extends continuously from $L^p(\mathbb{R}) \times L^q(\mathbb{R})$ *into* $L^r(\mathbb{R})$.

4. Simon Schulz, University of Cambridge

   email: sms79@cam.ac.uk

   Title: Linear degenerate ODEs arising in transonic flow

   Abstract: The objective of this talk is to present a method for solving certain linear degenerate ODEs that arise in the Morawetz problem for transonic flow. Specifically, these ODEs arise when solving for the Lax entropies associated to the system in question. We will start by introducing the transonic flow problem, and will then show existence for a particular family of such ODEs.

5. Eric Baer, University of Chicago

   email: ebaer@uchicago.edu

   Title: Stability results for weak Hessian determinants

   Abstract: In this talk we describe a program of work studying continuity results for the action of the Hessian determinant on spaces of Besov type into the space of distributions on $\mathbb{R}^N$, obtained in collaboration with D. Jerison. A key ingredient in the analysis is the characterization of certain Besov spaces as the space of traces of functions in Sobolev spaces. Other important ideas, arising in the context of showing optimality of our stability results, include the construction of suitable "atoms" having a tensor product structure and Hessian determinant of uniform sign, formation of lacunary series of rescaled atoms, and delicate estimates of terms in the resulting multilinear expressions. We conclude by discussing some results related to quantitative aspects of the classical weak stability of the Hessian determinant mapping.

6. Ryan Hynd, University of Pennsylvania

   Title: Asymptotic flatness of Morrey extremals

   Abstract: We will show that extremals for Morrey's inequality have a limit at infinity and also explain how to take a step in quantifying this convergence.

   email: rhynd@math.upenn.edu

7. Daesung Kim, University of Illinois at Urbana Champaign

   email: daesungk@illinois.edu

   Title: Deficit bounds for the log Sobolev inequality

   Abstract: The logarithmic Sobolev inequality, which states that the Fisher information is bounded below by the relative entropy, has been extensively studied in analysis and probability. In the Euclidean space, equality holds if and only if a measure is Gaussian. After the equality case is fully characterized by E. Carlen, we are interested in measuring how far a measure is away from Gaussian measures when it is close to achieving the equality. In this talk, we discuss a recent progress on this question for the logarithmic Sobolev inequality.

8. Cristian Gavrus, Johns Hopkins University

   email: cgavrus1@jhu.edu

   Title: Instability of the soliton for the focusing, mass-critical generalized KdV equation

   Abstract: In this talk we will discuss recent joint work with Benjamin Dodson. We discuss the proof of instability of the soliton for the focusing, mass-critical generalized KdV equation. This means that the solution to the generalized KdV equation for any

initial data with mass smaller than the mass of the soliton and close to the soliton in L2 norm must eventually move away from the soliton.

9. Matthew Jacobs, University of California, Los Angeles

   email: majaco@math.ucla.edu

   Title: Darcy's law with a source term

   Abstract: In this talk I will introduce a novel variant of the JKO scheme to approximate Darcy's law with a pressure dependent source term. By introducing a new variable that implicitly controls the source term, the scheme is still able to use the standard Wasserstein-2-metric even though the total mass changes over time. Leveraging the dual formulation of our scheme, we show that the discrete-in-time approximations satisfy many useful properties expected for the continuum solutions, such as a comparison principle and uniform L1-equicontinuity. Finally, I'll show that our discrete approximations converge to a solution of the corresponding PDE system, including a tumor growth model with a general nonlinear source term.

   Joint work with Inwon Kim and Jiajun Tong.

10. Alpar Meszaros, Durham University

    email: alpar.r.meszaros@durham.ac.uk

    Title: Degenerate nonlinear parabolic equations with discontinuous diffusion coefficients

    Abstract: Motivated by some physical and biological models, in this talk we consider a class of degenerate parabolic equations. Our analysis is based on gradient flows in the space of probability measures equipped with the distance arising in the Monge-Kantorovich optimal transport problem. The associated internal energy functionals in general fail to be differentiable, therefore classical results do not apply in our setting. We will study the combination of both linear and porous medium type diffusions and we show the existence and uniqueness of the solutions in the sense of distributions in suitable Sobolev spaces. Our notion of solution allows us to give a fine characterization of the emerging 'critical regions', observed previously in numerical experiments. A link to a three phase free boundary problem is also pointed out. It is possible to consider singular limits of our PDEs in a suitable way, to recover further degenerate models from the literature. The presented results have been obtained in collaboration with Dohyun Kwon (Madison).

11. Marco Cirant, University of Padova

    email: cirant@math.unipd.it

    Title: On maximal regularity for viscous Hamilton-Jacobi equations

Abstract: I will discuss some results on the regularity of solutions to semi-linear elliptic and parabolic equations of Hamilton-Jacobi type. I will in particular address the problem of "maximal regularity", conjectured some years ago by P.-L. Lions, and discuss two approaches based on the Bernstein method and duality.

Joint work with A. Goffi (Padova).

12. Alessandro Goffi, University of Padova

    email: agoffi@math.unipd.it

    Title: New qualitative properties for viscosity solutions of fully nonlinear degenerate PDEs

    Abstract: The purpose of this talk is to give an overview of recent developments on qualitative properties of viscosity solutions to fully nonlinear degenerate equations of second order. We discuss strong maximum and minimum principles via a new notion of subunit vector fields for fully nonlinear operators that generalizes the classical definition given by Fefferman-Phong, showing the propagation of maxima of a viscosity subsolution to such degenerate equations along the trajectories of generalized subunit vector fields. This implies the strong maximum and minimum principles when the operator admits a family of subunit vector fields satisfying the Hörmander condition. In particular, we show how the results can be applied to a large class of fully nonlinear and quasi-linear subelliptic equations structured over Hörmander vector fields, such as those on Carnot groups and Grushin geometries. Finally, we discuss strong comparison principles and one-side Liouville theorems for such fully nonlinear degenerate equations. These are joint works with M. Bardi (Padova).

13. Xavier Ros-Oton, Universitat de Barcelona

    email: xros@ub.edu

    Title: Stable cones in the thin one-phase free boundary problem

    Abstract: We study homogeneous stable solutions to the thin (or fractional) one-phase free boundary problem. The problem of classifying stable (or minimal) homogeneous solutions in dimensions $n > 2$ is completely open. In this context, axially symmetric solutions are expected to play the same role as Simons' cone in the classical theory of minimal surfaces, but even in this simpler case the problem is open. The goal of this talk is to present some new results in this direction.

    On the one hand we find, for the first time, the stability condition for the thin one-phase problem. Quite surprisingly, this requires the use of "large solutions" for the fractional Laplacian, which blow up on the free boundary.

    On the other hand, using our new stability condition, we show that any axially symmetric homogeneous stable solution in dimensions $n < 6$ is one-dimensional, independently of the parameter s between 0 and 1.

14. Serena Dipierro, The University of Western Australia

    email: serena.dipierro@uwa.edu.au

    Title: Boundary behaviour of nonlocal minimal surfaces

    Abstract: In this talk we present a peculiar behaviour of nonlocal minimal surfaces (i.e. local minimisers of a nonlocal perimeter functional), namely the capacity, and the strong tendency, of adhering to the boundary of the reference domain. This characteristic is in contrast not only with the boundary behaviour of classical minimal surfaces but also with the pattern produced by solutions of linear equations. We will discuss this phenomenon and present some recent results.

15. Stefania Patrizi, University of Texas at Austin

    email: spatrizi@math.utexas.edu

    Title: From the Peierls-Nabarro model to the equation of motion of the dislocation continuum

    Abstract: We consider a semi-linear integro-differential equation in dimension one associated to the half-Laplacian. This model describes the evolution of phase transitions associated to dislocations whose solution represents the atom dislocation in a crystal. The equation comprises the evolutive version of the classical Peierls-Nabarro model. We show that for a large number of dislocations, the solution, properly rescaled, converges to the solution of a well known equation called "the equation of motion of the dislocation continuum". The limit equation is a model for the macroscopic crystal plasticity with density of dislocations. In particular, we recover the so called Orowan's law which states that dislocations move at a velocity proportional to the effective stress. This is a joint paper with Tharathep Sangsawang.