Topology Qualifying Exam, Winter 2020

1. Let X and Y be spaces with Y Hausdorff. Let $f : X \longrightarrow Y$ be continuous and let $G = \{(x, f(x)) | x \in X\} \subset X \times Y$. Show G is closed.

2. Let $A \subset X$ be connected. Show $\bar{A}$ is connected.

3. Let X be a countable product of $\mathbb{R}$ topologized with the box topology and Y be a countable product of $\mathbb{R}$ topologized with the product topology. Prove X is not homeomorphic to Y.

4. Let X be a locally compact Hausdorff space and let Z be a subset with the property that $Z \cap K$ is closed for every compact $K \subseteq X$. Prove Z is closed.

5. Let $p : X \longrightarrow Y$ be a covering space and assume $\pi_1(Y)$ is abelian. Prove $\pi_1(X)$ is abelian.
Hint: Use homotopy lifting to show $p_* : \pi_1(X) \longrightarrow \pi_1(Y)$ is injective.

6. Let x_0 and x_1 be in the same path component of X. Construct an isomorphism from $\pi_1(X, x_0)^{ab}$ to $\pi_1(X, x_1)^{ab}$. If your construction involves on choices of paths, show that the isomorphism does not depend on these choices.

7. Let A denote the torus minus a point. Let B denote the sphere minus three points.
a) Are A and B homotopy equivalent?
b) Are A and B homeomorphic?