# MA 571 Qualifying Exam. January 2019. Professor McClure.

Each problem is worth 14 points and you get two points for free.

**Please be careful that your handwriting is clear and easy to read.**

**Unless otherwise stated, you may use anything in Munkres's book—but be careful to make it clear what fact you are using.**

**When you use a set theoretic fact that isn't stated in Munkres and isn't obvious, be careful to give a clear explanation.**

1. Let $X$ and $Y$ be topological spaces and suppose that $X$ is locally connected. Let $h : X \to Y$ be a function with the property that the restriction of $h$ to each component of $X$ is continuous. **Prove** that $h$ is continuous.

2. Let $X$ be a topological space and let $A$ be a connected subspace of $X$. **Prove** that $\bar{A}$ is connected. (This is a special case of a theorem in Munkres).

3. Let $X$ be a topological space and let $Y$ be a compact topological space. Let $\pi_1 : X \times Y \to X$ be the projection. Let $W \subset X \times Y$ be open. **Prove** that the set $S = \{x \in X \mid \pi_1^{-1}(x) \subset W\}$ is open.

4. Let $X$ be a compact topological space and let $\sim$ be an equivalence relation on $X$ with the property that $X/{\sim}$ is Hausdorff. Let $\sim'$ be the equivalence relation on $X \times [0,1]$ defined by $(x,t) \sim' (x_1,t_1) \Leftrightarrow x \sim x_1$ and $t = t_1$. **Prove** that $(X \times [0,1])/{\sim'}$ is homeomorphic to $(X/{\sim}) \times [0,1]$.

5. Let $X$ and $Y$ be topological spaces and let $h : X \to Y$ be a continuous function which induces the trivial homomorphism of fundamental groups. Let $x_0, x_1 \in X$ and let $f$ and $g$ be paths from $x_0$ to $x_1$. **Prove** that $h \circ f$ and $h \circ g$ are path homotopic.

6. Let $X$ be $\mathbb{R}^3$ with the $z$-axis removed, and let $x_0$ be the point $(1,0,0)$. What is $\pi_1(X, x_0)$? **Prove** that your answer is correct. You can just write down the formula for any deformation retraction that you use, you don't have to prove that it's continuous.

7. Let $p : E \to B$ be a covering map. Let $b_0 \in B$ and let $U$ be an evenly covered neighborhood of $b_0$. **Prove** that $p^{-1}(b_0)$ (considered as a subspace of $E$) is discrete.